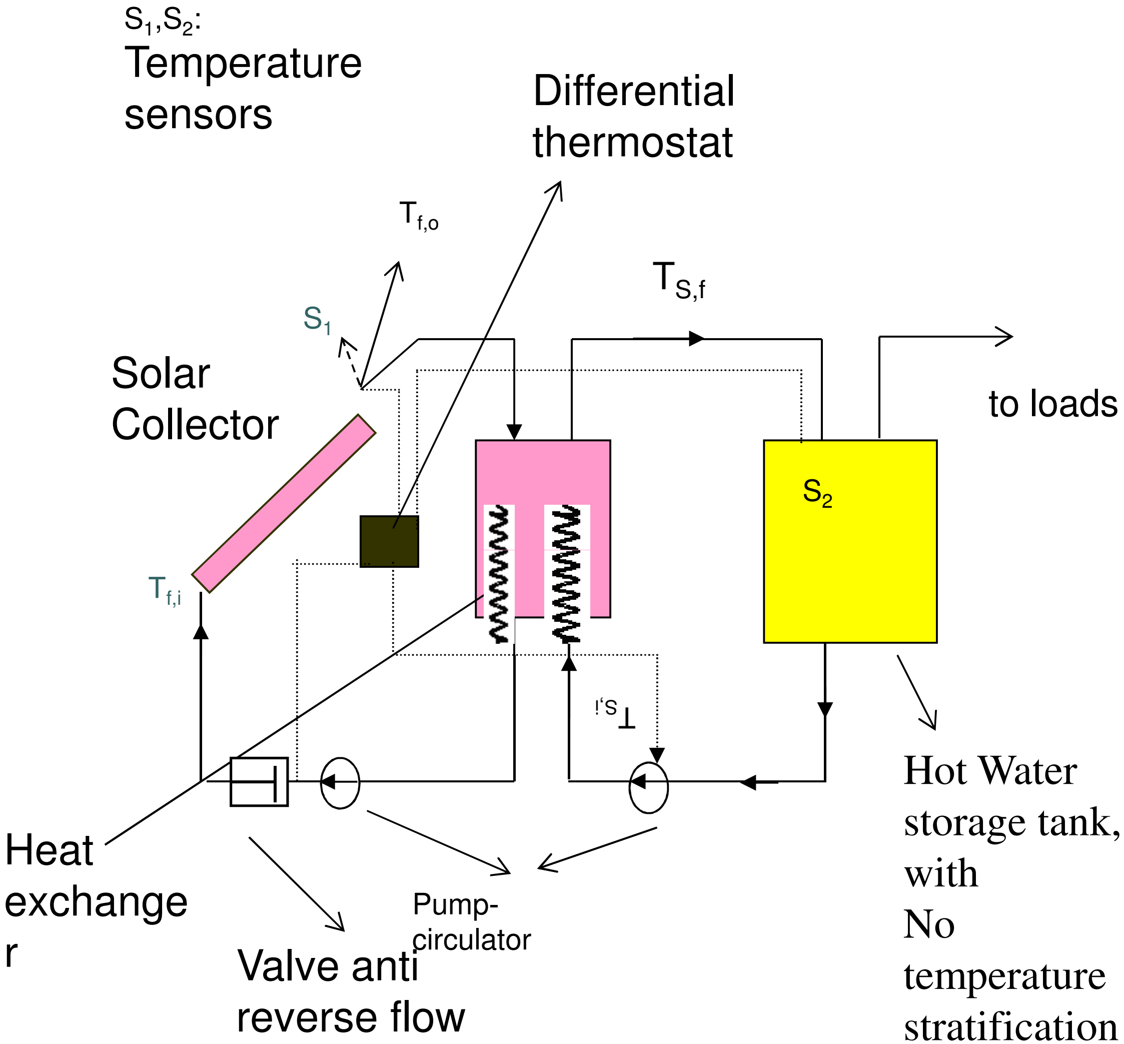
Basics of thermal analysis of solar collector
Simulation of operation of solar systems to determine the Thermal Gain.



The useful thermal energy Qu can be calculated from the relation:

$$Q_u = (m\,C_p)_f\,(T_{f,o} - T_{f,i}) \qquad (1.1)$$

or normalized to solar collector surface, the thermal power output is given by:

$$\dot{Q}_u = (\dot{m}\,C_p)_f\,(T_{f,o} - T_{f,i}) \qquad (1.2)$$

It can be also shown that the useful thermal gain from a solar collector is given by:

$$\dot{Q}_u = F_R A_c [I_T (\tau\alpha) - U_L (T_{f,i} - T_{\alpha})] \qquad (1.3)$$

A similarity to the previous expressions holds for the hot water tank,too:

$$\dot{Q}_u = (\dot{m} C_p)_s \cdot (T_{s,f} - T_{s,i}) \qquad (1.4)$$

or equivalently

$$\dot{Q}_u = (\dot{m} C_p)_{min} \cdot \varepsilon \cdot (T_{f,o} - T_{s,i}) \qquad (1.5)$$

The coefficient of effectiveness, ε, is given by an expression in any Heat Transfer book,

$$\varepsilon = \frac{Q}{Q_{max}} = \frac{(\dot{m}C_p)_c(T_{f,o} - T_{f,i})}{(\dot{m}C_p)_{min}(T_{f,o} - T_{s,i})} = \frac{1 - e^{-NTU(1-C)}}{1 - C \cdot e^{-NTU(1-C)}} \qquad (1.6)$$

From the expression (1.2) of calorimetry one gets:

$$C = (\dot{m}C_p)_{min}/(\dot{m}C_p)_{max} \qquad NTU = (U_A \cdot A)_{\varepsilon\nu}/(\dot{m}C_p)_{min}$$

$$T_{f,o} = T_{f,i} + \frac{\dot{Q}_u}{(\dot{m}\ C_p)_f} \qquad (1.7)$$

Substitute $T_{f,i}$ from (1.7) to (1.3), then the equation which provides the Thermal Power stored in the system-tank takes the form:

$$\dot{Q}_u = \frac{A_c F_R \left[I_T (\tau\alpha) - U_L \left(T_{f,o} - T_\alpha \right) \right]}{1 - \dfrac{A_c F_R U_L}{(\dot{m} C_p)_f}} \tag{1.8}$$

We solve eq. (1.5) for $T_{f,o}$ and we get:

$$T_{f,o} = \frac{\dot{Q}_u}{(\dot{m} C_p)_{min} \cdot \varepsilon} + T_{s,i} \tag{1.9}$$

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References

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Substitution of $T_{f,o}$ to (.1.8), gives:

$$\dot{Q}_u = \frac{A_c F_R \left[I_T(\tau\alpha) - U_L \cdot \left[\frac{\dot{Q}_u}{(\dot{m} C_p)_{min} \cdot \varepsilon} + T_{s,i} - T_\alpha \right] \right]}{1 - \frac{A_c \cdot F_R U_L}{(\dot{m} C_p)_f}} \qquad (1.10)$$

The expression (1.10) can be easily simplified to:

$$\dot{Q}_u = \frac{A_c F_R \left[I_T(\tau\alpha) - U_L (T_{s,i} - T_\alpha) \right]}{1 + \frac{A_c F_R U_L}{(\dot{m} C_p)_f} \times \left[\frac{(\dot{m} C_p)_f}{(\dot{m} C_p)_{min} \cdot \varepsilon} - 1 \right]} \qquad (1.11)$$

We define a new parameter, F'_R:

$$F'_R = \frac{F_R}{1 + \frac{A_c F_R U_L}{(\dot{m} C_p)_f} \times \left[\frac{(\dot{m} C_p)_f}{(\dot{m} C_p)_{min} \cdot \varepsilon} - 1 \right]} \tag{1.12}$$

Hence, the expression (1.11) is simplified as to:

$$\dot{Q}_u = A_c * F'_R \left[I_T (\tau\alpha) - U_L (T_{s,i} - T_\alpha) \right] = F_R A_c * \frac{F'_R}{F_R} \left[I_T (\tau\alpha) - U_L (T_{s,i} - T_\alpha) \right] \tag{1.13}$$

Let us consider a small time period the solar collector system operates. Then, the mean water temperature in the storage tank is determined by:

$$T_s = \frac{T_{s,i} + T_{s,f}}{2} \tag{1.14}$$

The heat delivered by a collector, Ac, in a period, Δτ, to the tank may be determined by:

$$Q_{u,\Delta\tau} = (MC_p)_s (T_{s,f} - T_{s,i}) = \int_{\tau}^{\tau+\Delta\tau} \dot{Q}_u dt \tag{1.15}$$

We divide both sides of (.1.15) over Ac to normalize the expression. Then

$$q_{u,\Delta\tau} = (m\,C_p)_s\,(T_{s,f} - T_{s,i}) \qquad (1.16)$$

or equivalently

$$T_{s,f} - T_{s.i} = q_{u,\Delta\tau}/(m\,C_P)_s \qquad (1.17)$$

Substitute Ts,f from (1.17) to (1.14). We get:

$$T_s = T_{s,i} + \frac{q_{u,\Delta\tau}}{(2\ m\ C_P)_s} \tag{1.18}$$

T_s is the mean temperature of the water in the tank in the above time interval. Integration of (1.13 or 1.15) for this time period gives:

$$Q_{u,\Delta\tau} = F_R' \times A_c[H_h\overline{(\tau\alpha)} - U_L(\overline{T_s} - T\alpha)\Delta\tau] \tag{1.19}$$

Substitute Ts from (1.18) to (1.19). We get:

$$q_{u,\Delta\tau} = \frac{F_R'[H_n(\overline{\tau\alpha}) - U_L(T_{s,i} - \overline{T_\alpha})\Delta\tau}{1 + \frac{F_R' U_L}{2(m\,C_P)_s} \times \Delta\tau} \tag{1.20}$$

Let us analyze a real case

A Solar Collector System, as the one shown in the 1st figure, has parameters: $F`_R U_L$=3.5 W/m^2*K and $F`_R(\tau\alpha)_n$= 0.69

and is placed at horizontal position in Pyrgos.

The storage tank has capacity 50 l/m^2.

Let the storage tank temperature at 7:30 be 20 C.

Please determine the hourly temperature in the tank and the hourly efficiency.

Data input: ambient temperature , T_a, and the mean hourly global solar radiation, H_n. Values are given in the Table below

Data input and values of basic quantities as provided by the iteration procedure to be outlined below.

Time 18.4.1999	T_{α} °C	H_n $(\frac{kJ}{m^2})$	q_u $(\frac{kJ}{m^2})$	$T_{s,i}$ °C	$T_{s,f}$ °C	$N=q_u/H_n$
7:30 – 8:30	15.0	720	421	20	22	0.58
8:30 – 9:30	15.5	1476	909	22	26	0.61
9:30 – 10:30	16.5	1980	1206	26	32	0.60
10:30 – 11:30	17.0	2484	1479	32	39	0.59
11:30 – 12:30	17.5	2844	1640	39	46	0.57
12:30 – 13:30	18.0	3240	1816	46	55	0.56
13:30 – 14:30	19.0	3250	1729	55	63	0.53
14:30 – 15:30	19.0	2968	1439	63	70	0.48
15:30 – 16:30	18.0	2412	971	70	75	0.40
16:30 – 17:30	17.5	1800	498	75	77	0.27
17:30 – 18:30	17.0	1210	68	77	78	0.05

To determine the quantities $T_{s,f}$,$T_{s.i}$

A. Time Interval 7:30 – 8:30 am

Step 1st : We determine the normalized useful heat by (1.20).

$$\mathbf{q_{u,\Delta\tau}} = \frac{720\text{kJ}/\text{m}^2 \times 0.69 - 3.5\dfrac{\text{W}}{\text{m}^2\text{K}} \times (20^0 - 15^0) \times 3600\text{s}}{[1 + \dfrac{3.5\dfrac{\text{W}}{\text{m}^2 K} \times 3600\text{s}}{2 \times 50\dfrac{\text{kg}}{\text{m}^2} \times 4180\dfrac{\text{J}}{\text{kg K}}}] = 1.03} = \mathbf{421}\text{kJ}/\text{m}^2 \qquad (1.21)$$

Step 2nd : We determine tank temperature, $T_{s,f}$, at the end of the 1st interval 8:30 amusing expression (1.17)

$$\mathbf{T_{s,f}} = 20^0\mathrm{C} + \frac{421{,}000\mathrm{J/m^2}}{50\,\mathrm{kg/m^2} \times 4180\mathrm{J/kg\,m^2}} = \mathbf{22}\ {}^0\mathrm{C} \qquad (1.22)$$

Step 3rd : Determine efficiency, η, during this short period by :

$$\boldsymbol{\eta} = \frac{\mathbf{Q_{u,\Delta\tau}}}{\mathbf{A_c H_n}} = \frac{\mathbf{q_{u,\Delta\tau}}}{\mathbf{H_n}} \qquad \boldsymbol{\eta} = \frac{\mathbf{q_{u,\Delta\tau}}}{\mathbf{H_n}} = \frac{421\mathrm{kJ/m^2}}{720\mathrm{kJ/m^2}} = \mathbf{0.58} \qquad (1.23)$$

B. Time Interval 8:30 – 9:30am

We follow the same procedure as before. We put for this period 8:30-9:30 as $T_{s,i}$, the $T_{s,f}$ value of the previous interval.

Determine $\mathbf{q}_{\mathbf{u},\ \Delta\tau}$ from (1.20)

$$\mathbf{q}_{\mathbf{u},\Delta\tau} = \frac{1476\mathrm{kJ/m^2} \times 0.69 - 3.5\dfrac{\mathrm{W}}{\mathrm{m^2\,{}^0K}} \times (22^0 - 15.5^0) \times 3600\mathrm{s}}{1.03} = \mathbf{909}\mathrm{kJ/m^2}$$

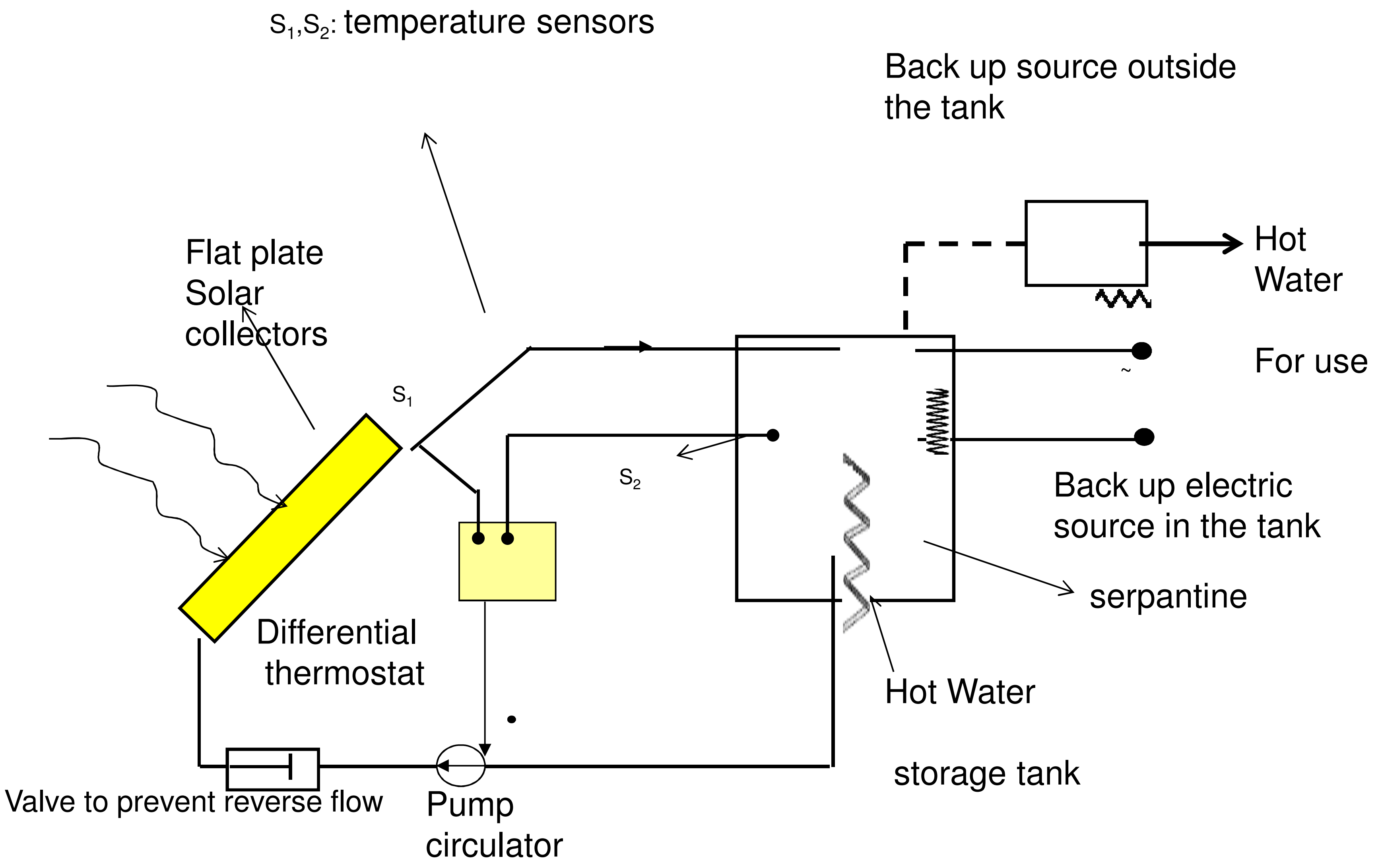
(1.24)

Determine $T_{s,f}$ from (1.17)

$$T_{s,f} = 22\,^0\mathrm{C} + \frac{909000 \ \frac{\mathrm{J}}{\mathrm{m}^2}}{50 \ \frac{\mathrm{kg}}{\mathrm{m}^2} \times 4180 \ \frac{\mathrm{J}}{\mathrm{kg}\,^o\mathrm{C}}} = \mathbf{26}\ ^0\mathrm{C} \qquad (1.25)$$

Then, the efficiency is estimated by:

$$\boldsymbol{\eta = \frac{q_{u,\Delta\tau}}{H_n} = \frac{909 \ kJ/\ m^2}{1476 \ kJ/\ m^2} = 0.61} \qquad (1.26)$$



S_1,S_2: temperature sensors
Back up source outside the tank
Flat plate Solar collectors
Hot Water
For use
S_1
S_2
Back up electric source in the tank
serpantine
Differential thermostat
Hot Water
storage tank
Valve to prevent reverse flow
Pump circulator

2. **A generalized analysis to consider the Load,** too.

Let us consider q_s as the net stored heat normalized to collector surface; that is when the Load, Q_L, is subtracted. Correspondingly, the thermal load per collector surface is denoted by, ($q_L = Q_L / Ac$). Then it holds :

$$q_S = q_{u,\Delta\tau} - q_L \qquad q_{u,\Delta\tau} = q_S + q_L \tag{2.1}$$

Following the procedure as for expression (1.18) there is given that:

$$T_s = T_{s,i} + \frac{q_s}{2(m C_P)_s} \tag{2.2}$$

2. A generalized analysis to consider the Load, too.

Substitute Ts to (1.19). Then, the expression (1.20) is modified and due to (2.1) we get :

$$q_s = \frac{F'_R \left[H_n(\overline{\tau\alpha}) - U_L(T_{s,i} - \overline{T_\alpha})\Delta\tau\right] - q_L}{1 + \dfrac{F'_R U_L}{2(mC_P)_s}\Delta\tau} \qquad (2.3)$$

We substitute (2.3) in (2.1) and we finally get the generalized iterative formula:

$$q_{u,\Delta\tau} = \frac{F'_R[H_n(\overline{\tau\alpha}) - U_L(T_{s,i} - \overline{T\alpha})\Delta\tau}{1 + \dfrac{F'_R U_L}{2(mC_p)_s}\Delta\tau} + \frac{q_L}{1 + \dfrac{2(mC_p)_s}{F'_R U_L \Delta\tau}} \qquad (2.4)$$

References

Renewable Energy Systems: Theory and Intelligent Applications
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Nova Science Publishers, N.Y., 2012