Stochastic prediction of hourly global solar radiation profiles

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Abstract

- A stochastic prediction model of the hourly profile of the $I(h;n_j)$ for any day n_j at a site is outlined.
- It requires 1,2,or 3 measurements of the global solar radiation in a day n_j, uses a D.B. and gives $I(h;n_j)$ for the rest hours.
- The model is validated against solar measurements.
- Conclusions are deducted for the predictive power of the model developed in MATLAB
- It provides $I(h;n_j)$ profile predictions very close to the measured values and
- offers itself as a promising tool for a predictive on-line daily load management.

Introduction

- The prediction of the global solar radiation $I(h;n_j)$ on an hourly, h, basis, for any day, n_j, was the target of many attempts internationally.
- Review papers outline and compare the **mean expected, $I_{m,exp}(h;n_j)$,** values which are the results of several such significant approaches.
- A reliable methodology **to predict the $I(h;n_j)$ profile,** based on few data, taking into account morning measurement(s) and simulating the statistics of $I(h;n_j)$, is a challenge.

- A model to predict close to reality I(h;nj) values would be useful in problems such as:

1. Meteo purposes
2. Sizing effectively and reliably the solar power systems i.e. PV generators
3. Management of solar energy sources, i.e. the output of the PV systems, as affected by the meteo conditions in relation to the power loads to be met .

- The above issues drive the research activities towards the development of an improved effective methodology to predict the $I(h;n_j)$ for any day, n_j, of the year at any site with latitude φ.

- One of those methodologies to predict the mean expected hourly global solar radiation, as proposed by the authors, provided a simple approach model based on the function:
-

$$I(h;n_j) = a + b \cdot \cos(2 \cdot \pi \cdot h/24) \quad (1)$$

where, α and b are constants, which depend on the day nj and the site φ,

- This model, overestimates $I(h;n_j)$ at the early morning and late afternoon hours, while it underestimates $I(h;n_j)$ around the solar noon hours.
- Although, the $I_{pr}(h;n_j)$ values fall, in general, within the range of the measured $I_{mes}(h;n_j)$ fluctuations, a more accurate and dynamic model had to be developed.
- That model should have inbuilt statistical fluctuations, like the METEONORM package, but with a more effective prediction power.

- A comparison of the $I_{pr}(h;n_j)$ values between METEONORM and the generalised model versions, outlined, in contrast to measured $I_{mes}(h;n_j)$ values, during the years (1995-2000), will be highlighted.

Basic Theoretical Analysis

- Due to the mentioned drawback of the simple model , a correction factor is introduced, normalized at solar noon

$$\frac{e^{\left(-\mu(n_j)\cdot x(\theta_z)\right)}}{e^{\left(-\mu(n_j)\cdot x(\theta_z;\omega=0)\right)}}$$

$$\mu(n_j) = \frac{-\ln\left(\dfrac{H(n_j)}{H_{ext}(n_j)}\right)}{x_m(\theta_Z)}$$

$$H(n_j) = A + B \cdot \cos\left((2 \cdot \pi / 364) \cdot n_j + C \right)$$

Zone	1	2	3	4	5	6
A	16.607	15.862	15.140	14.892	14.283	13.742
B	9.731	9.305	10.326	9.650	9.290	8.950
C	9.367	9.398	9.405	9.394	28.265	78.535

- $x(\theta_z)$ is the distance the solar beam travels within the atmosphere and $x(\theta_z;\omega=0)$ is the distance at solar noon, for which $\omega=0$. Notice that,

$$x(\theta_Z) = -R_g \cdot \cos(\theta_Z) + \sqrt{R_g^2 \cdot \cos^2(\theta_Z) + (R^2 - R_g^2)}$$

$$R = R_g + H_{atm}$$

where, Rg is the earth’s radius = 6.35·103 km, and H_{atm} the height of the atmosphere =2.5 km.

Also, $\cos(\theta_Z)$ is given by formula below, where δ is the solar declination and ω is the hour angle.

$$\cos(\theta_Z) = \cos(\varphi)\cdot\cos(\delta)\cdot\cos(\omega) + \sin(\varphi)\cdot\sin(\delta)$$

For ω =0 or h=12, at solar noon, we get:

$$\cos(\theta_{Z,0}) = \cos(\varphi)\cdot\cos(\delta) + \sin(\varphi)\cdot\sin(\delta) = \cos(\varphi - \delta)$$

$$\theta_{Z,0} = \varphi - \delta$$

Notice that, at sunset θ_z =90^0, and therefore, for sunsegett we :

$$x(\theta_Z = 90^\circ) = x(\omega_{ss}) = \sqrt{R^2 - R_g^2}$$

- $x_m(\theta_z)$ is the mean daily distance the solar beam travels in the atmosphere. It is determined by the formula:

$$x_m(\theta_Z) = \frac{\sum_{i=1}^{n} x(\theta_Z)_i}{N}$$

$$x_m(\theta_Z) = \frac{\sum \left(x(\theta_Z) \cdot I(h; n_j) \right)_i}{\sum I(h; n_j)_i}$$

More accurately, xm(θz) should be weighted over the solar intensity.
Finally, the model to predict the mean expected hourly global solar radiation now takes the form:

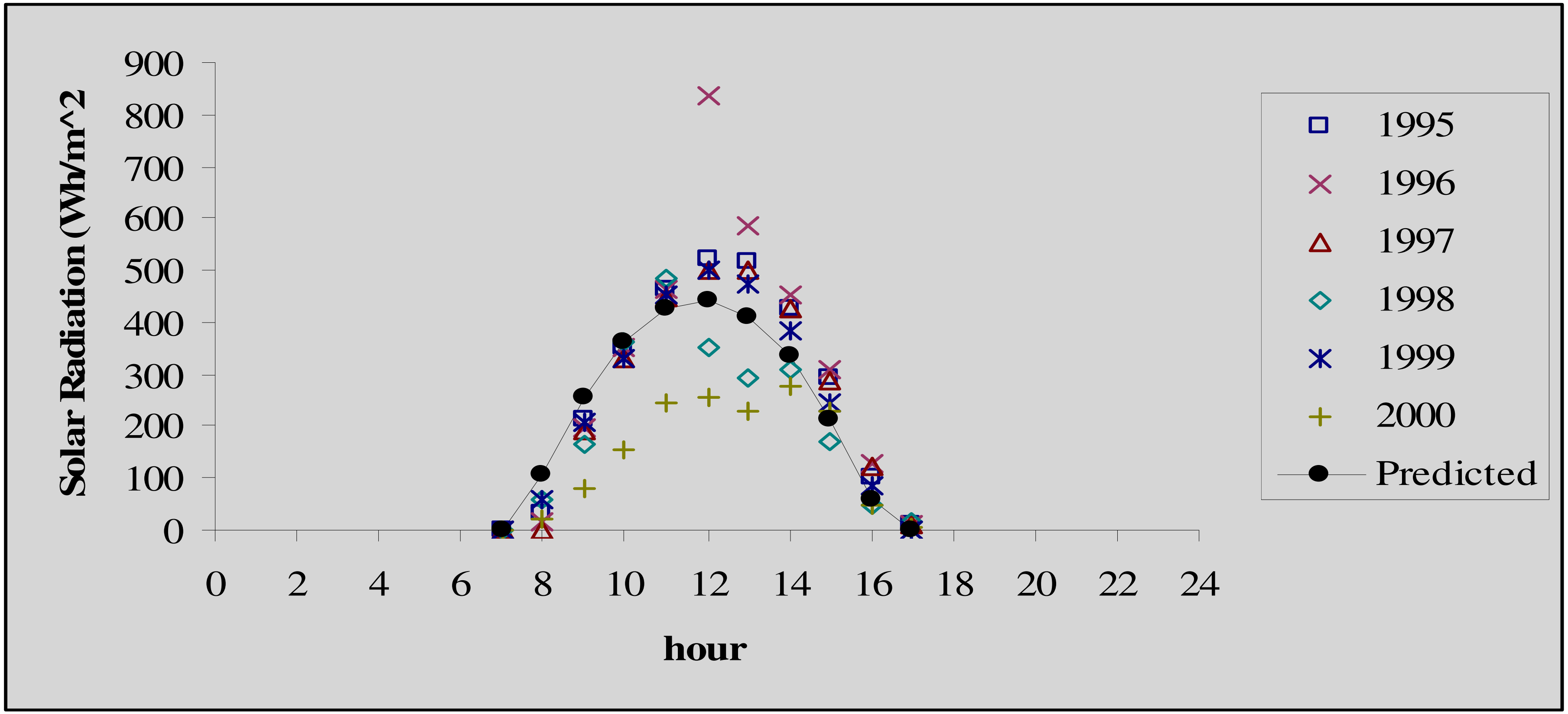
$$I(h;n_j) = \alpha + b \cdot \frac{e^{-\mu(n_j)\cdot x(h)} \cdot \cos\left(2\cdot\pi\cdot h/24\right)}{e^{-\mu(n_j)\cdot x(h=12)}}$$

α and b, are determined , using the boundary conditions to be highlighted below :

$I(h; n_j) = 0$ at $h = h_{ss}$, where $hss = 12h + \omega_{ss} / 15^0/h$

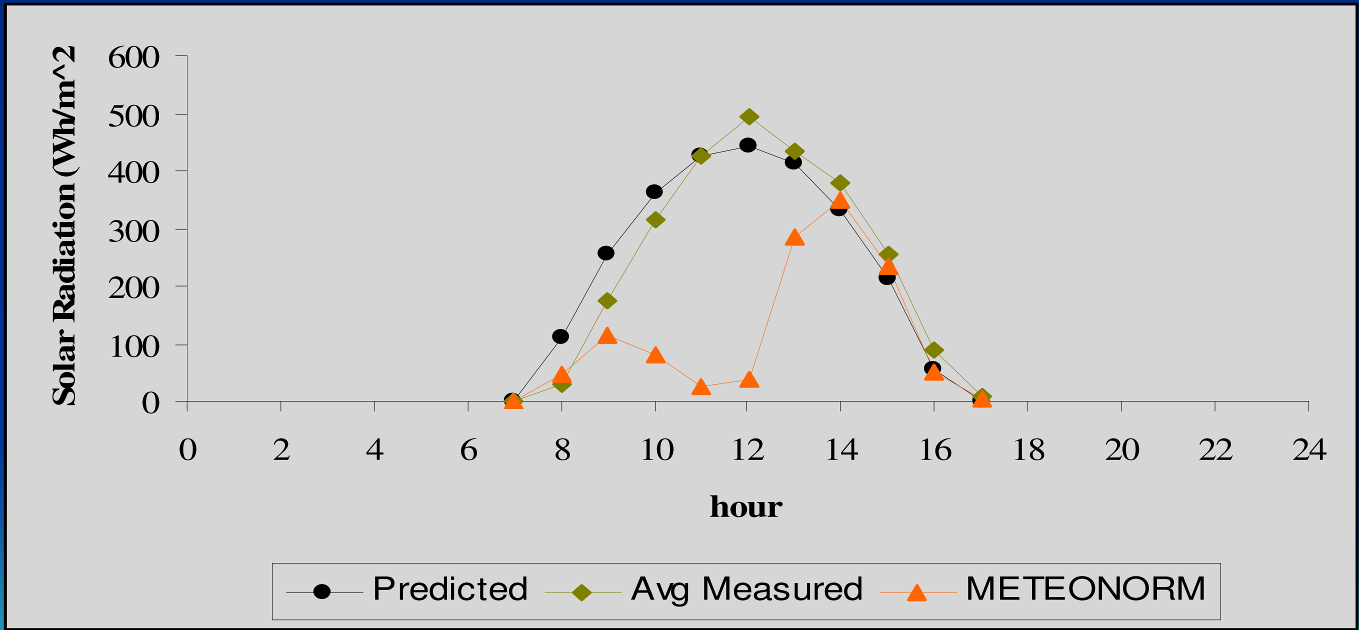
$$\omega_{ss} = \mathrm{acos}(-\tan(\varphi)\cdot\tan(\delta))$$

Integration of I(h,nj) over a day, from ω_{sr} to ω_{ss}, provides H(nj) and via these 2 boundary conditions we may determine a and b

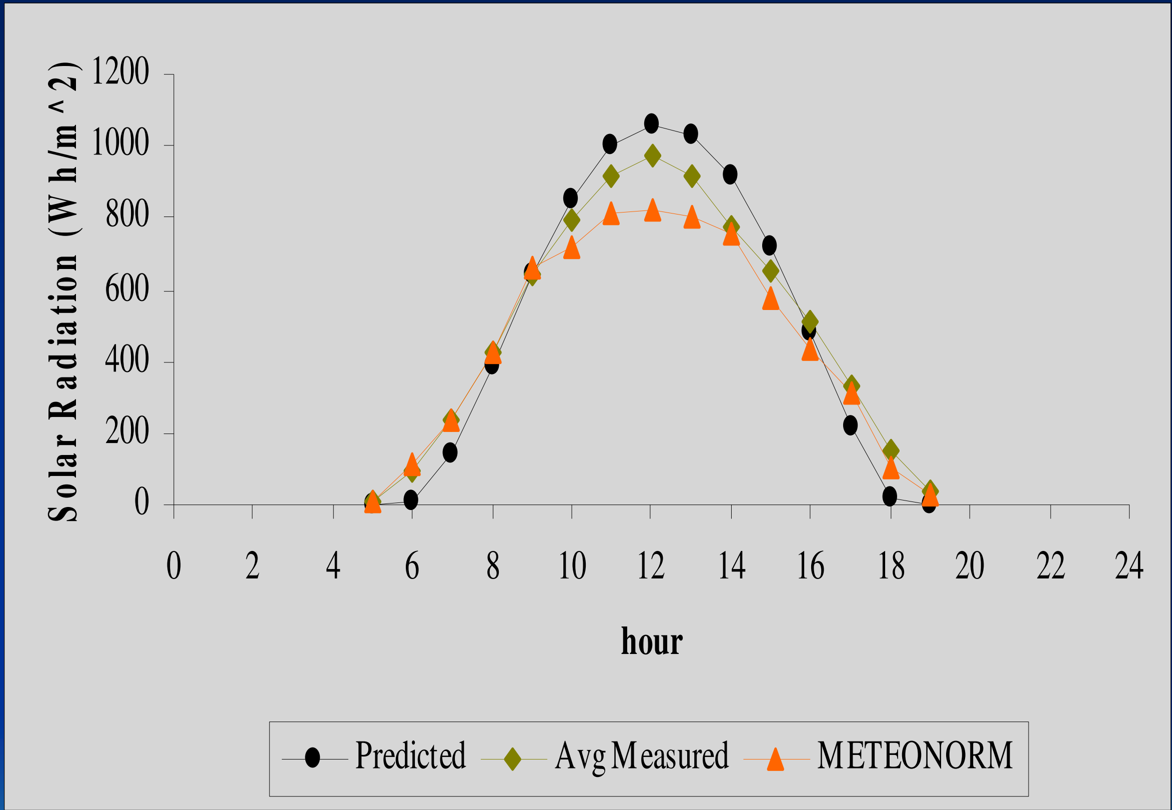


A comparison of the predicted mean $I_{m,pr}$(h; 17.1) and the measured I_{mes}(h; 17.1) values, during winter for Patra, Greece, for the years 1995-2000.

- The means of $I_{mes}(h;nj)$, the $I_{m,pr}(h;nj)$ values, as predicted by this model, and the $I_{MET}(h;n_j)$ values, with the inbuilt stochastic generator for fluctuations, provided by the METEONORM package for the days 17.1 and 17.7, are shown below.
- The results show the presence of expected strong fluctuations in $I(h;n_j)$ especially, for winter months.



Values of the mean $I_{mes}(h;17.1)$, $I_{m,pr}(h;17.1)$ and $I_{MET}(h;17.1)$ for Patra, Greece.



Values of the mean $I_{mes}(h;17.7)$, $I_{m,pr}(h;17.7)$ and $I_{MET}(h;17.7)$ for Patra, Greece.

$I(h;n_j)$ Predicted Values by the Stochastic Mode of this Model

- The prediction methodology, proposed, takes into account a first early $I(h; n_j)$ measurement, at hour h_1.
- The model to predict $I(h;n_j)$, as described in this paper, introduces a stochastic factor, which takes into account the previous hour $I(h-1;n_j)$ value.
- The steps followed to predict $I(h;n_j)$ based on a morning measurement, $I_{mes}(h;n_j)$, are outlined below:

- 1. Let $I_{m,pr}(h;nj)$ is predicted as outlined before. Such values are, easily, obtained using a MATLAB program and algorithms, developed for this research project.

- 2. For n_j=17, solar intensity measurement, at hour h_1, provides Imes(h_1;17), whose s.d. is σ_I
- 3. σ_I is pre-determined for morning hours [h_{sr} , h_{sr} +3], afternoon periods [h_{ss}-3, h_{ss}], as well as for the hours around solar noon.

- For example: Let the p.d.f. is a normal distribution with $\sigma/I*100\% = 25\%$ at morning hours.

- 4. Let us start with $I_{mes}(h_1;17)$. The program predicts $I_{m,pr}(h_1; 17)$ and subtracts it from the measured $I_{mes}(h_1; 17)$, for that hour, h_1. The result is compared to σ_I,

$$\frac{I_{mes}(h_1;17) - I_{m,pr}(h_1;17)}{\sigma_I} = \delta I/\sigma_I = \lambda$$

Possible values of the above expression for $\delta I/\sigma_I$, according to normal distribution may lie in the region: [-4, +4]. The interval of σ_I ,where I(h;nj) lies, in the first hour h_1, may be determined by equation above.

Let this deviation be $\lambda^*\sigma_I$.

- 5. An attempt is made to predict $I_{pr}(h_2;n_j)$, at $h_2=h_1+1$ hour.

  The model tries to give an estimate of the $I_{pr}(h_2;n_j)$, taking into account the deviation of $I_{mes}(h_1;17)$ from the mean expected value $I_{m,pr}(h_1;17)$, as determined in step 4.

  In this step, the model samples from a Gaussian p.d.f. in order to estimate in which σ_I' interval of the normal distribution, the $I(h_2;n_j)$ value lies. That is, it determines the new value of $\lambda^*\sigma_I$; let it be $\lambda'\cdot\sigma_I'$.

- 6.The predicted value of $I(h_2;n_j)$ is given, in this step, based on the mean expected $I_{m,pr}(h_2;17)$, with a new deviation value $\lambda'\cdot\sigma_I'$, i.e.

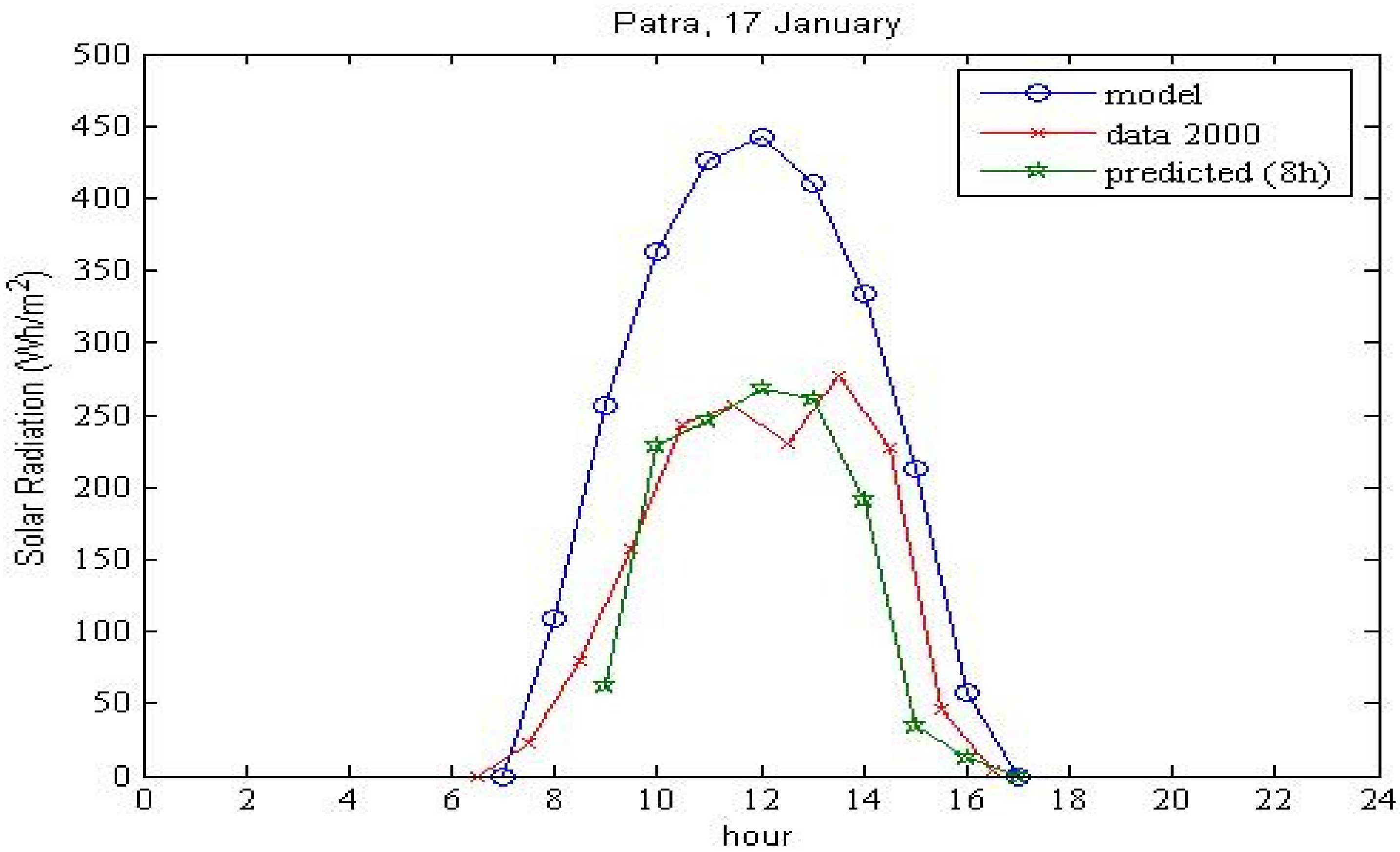
$$I_{pr}(h_2;nj)= I_{m,pr}(h_2;n_j) \pm \lambda'\cdot\sigma_I'$$

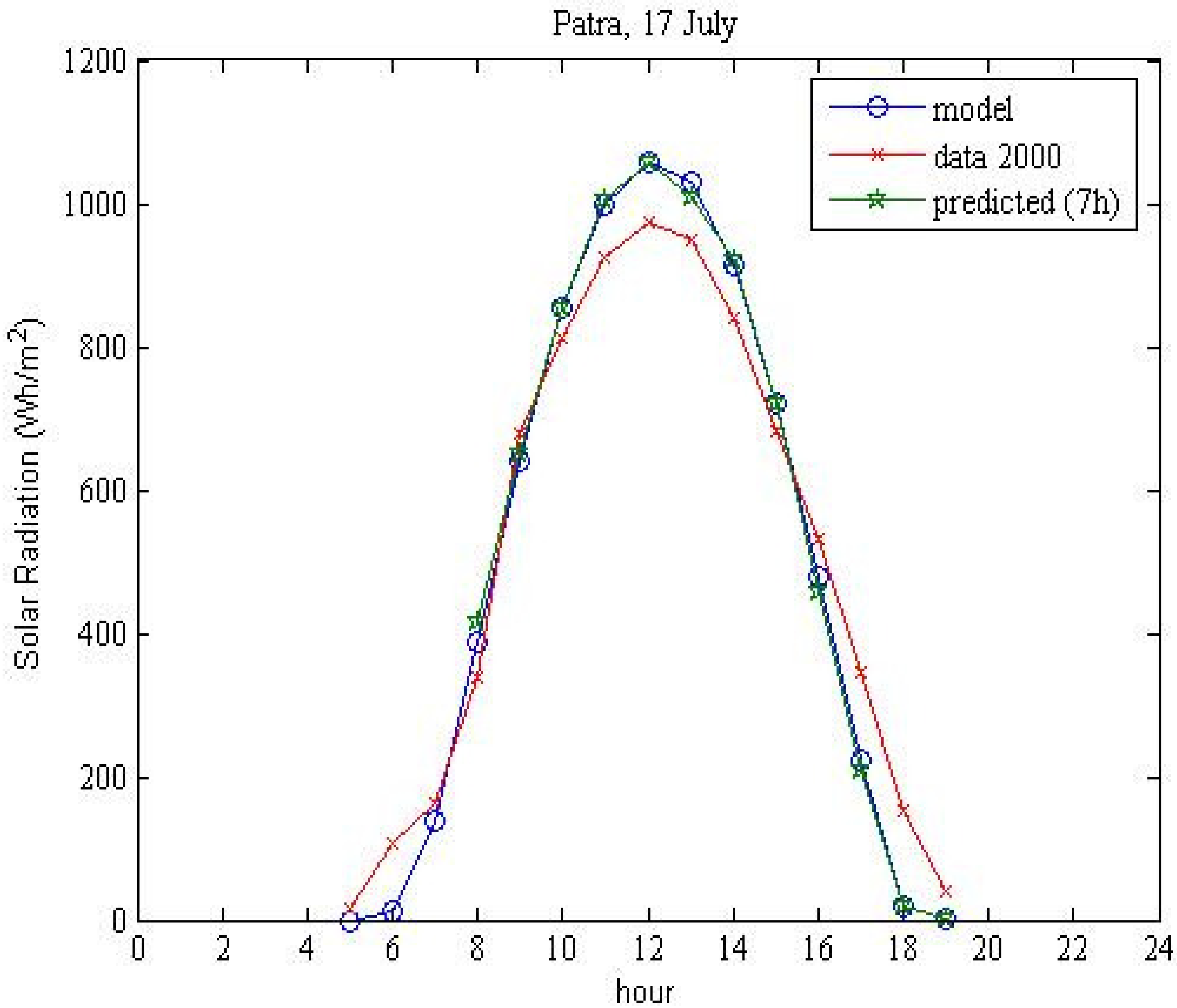
λ' is determined through a Gaussian sampling and is permitted to take, according to this model, values within the range $\lambda\pm1$.

7. The model, then, determines the $I_{pr}(h_2;n_j)$ value and compares it with the mean expected $I_{m,pr}(h_2;n_j)$.

It repeats the cycle, steps 4-6, for the h_3 hour and so on.

- It is important to note that if λ is as extreme as +3 or -3, the model requires that $I_{pr}(h;n_j)$ lies in the same σ_I region, for all day long, without jumping to other σ_I intervals.





Patra, 17 July
Solar Radiation (Wh/m^2)
hour
model
data 2000
predicted (7h)
0
200
400
600
800
1000
1200
0
2
4
6
8
10
12
14
16
18
20
22
24

From Mode I: I(h;nj) prediction based on one morning measurement to ***Mode II: I(h;nj) prediction based on two morning measurements***

- Mode II of this model considers for the $I(h;n_j)$ prediction two morning solar radiation measurements.
- In addition to Mode I, it takes into account the rate of change of the difference $[I_{meas}(h;n_j)- I_{av}(h;n_j)]$, during the period from h_1 to h_2.
- Conclusively, it includes two stochastic terms, one term which stands for the stochastic fluctuations at hour h_3, and
- a second term to stand for the rate of change of the $I(h;n_j)$, within the time interval $[h_1, h_2]$.

- Similarly, as in Mode I, the parameter t_2 is determined

$$t_2 = \frac{I_{meas}(h_2;n_j) - I_{av}(h_2;n_j)}{\sigma_{I(h_2;n_j)}}$$

Finally, the $I(h_3;n_j)$ value for the next time interval, h_3, is predicted by an improved expression:

$$I_{pr}(h_3;n_j) = I_{av}(h_3;n_j) + R\cdot\sigma_{I(h_3;n_j)} + \frac{1}{4}\cdot\left(t_2\cdot\sigma_{I(h_2;n_j)} - t_1\cdot\sigma_{I(h_1;n_j)}\right)\cdot R_1$$

- $\sigma_I(h_2;n_j)$ and $\sigma_I(h_3;n_j)$ are the s.d. of the measured $I(h;n_j)$ values at hours h_2 and h_3, respectively, in the day n_j, as obtained from the D.B.

- Furthermore, the model proceeds to predict the $I_{pr}(h_4;n_j)$ value. This is based on $I_{pr}(h_3;n_j)$, which is the previous hour, h_3, predicted value and the measured $I_{meas}(h_2;n_j)$ one.

- Further on, it may predict $I_{pr}(h_5;n_j)$, based on the previously predicted values $I_{pr}(h_3;n_j)$ and $I_{pr}(h_4;n_j)$, and so on.

- Notice that, R is a random number which may take values within $t_2 \pm 1$.
- This implies that from hour to hour, one may not expect weather variations larger than $\pm 1 * \sigma_I$
- R1 is randomly distributed according to a Gaussian p.d.f. (0, 1). The term $(t_2 * \sigma_I(h_2;n_j) - t_1 * \sigma_I(h_1;n_j)) * R1$ stands for the contribution to the I(h;nj) prediction by its rate of change during the 2 previous hours.

This contribution is estimated by the relative positions of the measured $I_{meas}(h_1;nj)$ and $I_{meas}(h_2;n_j)$, with reference to the average values $I_{av}(h_1;n_j)$ and $I_{av}(h2;nj)$, one by one.

- The above Formula expresses the superposition principle of two processes.

- The first one is the short term stochastic behaviour which provides expected hourly fluctuations based on the past history of the stored data for the hour h of a day n_j.

- The second one represents the present trend of the hourly $I(h;n_j)$ measurements during the interval h_1 to h_2. This trend is weighted over a Gaussian p.d.f., (0, 1), which underlines that the two processes, short and long term, are independent to each other, and this ensemble represents the real phenomenon.

Mode III: I(h;nj) prediction based on three morning measurements

- The Mode III of the proposed model takes into consideration 3 morning solar global radiation measurements for the prediction of $I(h;n_j)$.
- According to the concept presented earlier, the prediction of $I(h_4;n_j)$ at hour h_4 is based on the following formula, which is more advanced than the previous two.

$$I_{pr}(h_4;n_j) = I_{av}(h_4;n_j) + R\cdot\sigma_{I(h_4;n_j)} + \frac{1}{4}\cdot\left(t_3\cdot\sigma_{I(h_3;n_j)} - t_2\cdot\sigma_{I(h_2;n_j)}\right)\cdot R1 +$$

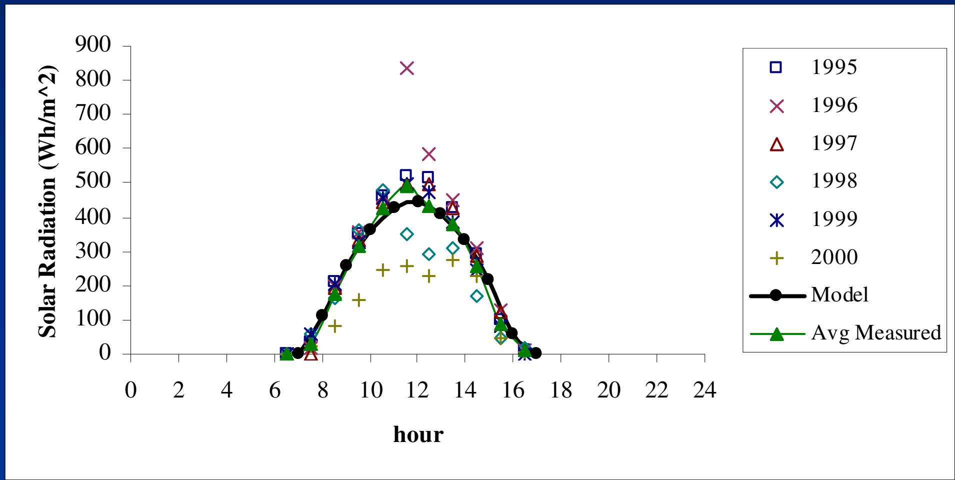
$$+\frac{1}{9}\cdot\left(t_3\cdot\sigma_{I(h_3;n_j)} - 2\cdot t_2\cdot\sigma_{I(h_2;n_j)} + t_1\cdot\sigma_{I(h_1;n_j)}\right)\cdot R2$$

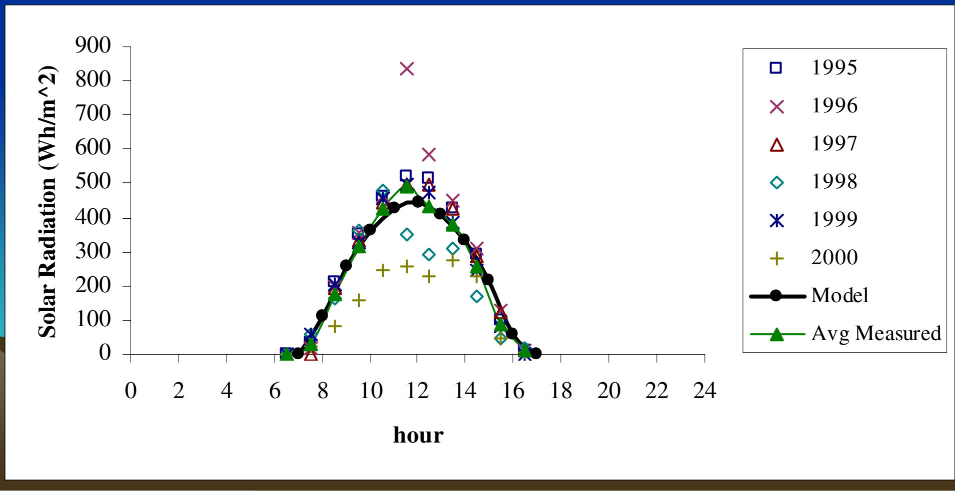
In eq above there appears a 3^{rd} and 4^{th} term .
The 3^{rd} term gives a measure of the rate of change of $[I_{meas}(h;nj)- I_{av}(h;nj)]$, during the two hours, $[h_2 , h_3]$, prior to the hour, h_4

The fourth term gives the rate of change of the above difference during the three previous hours.

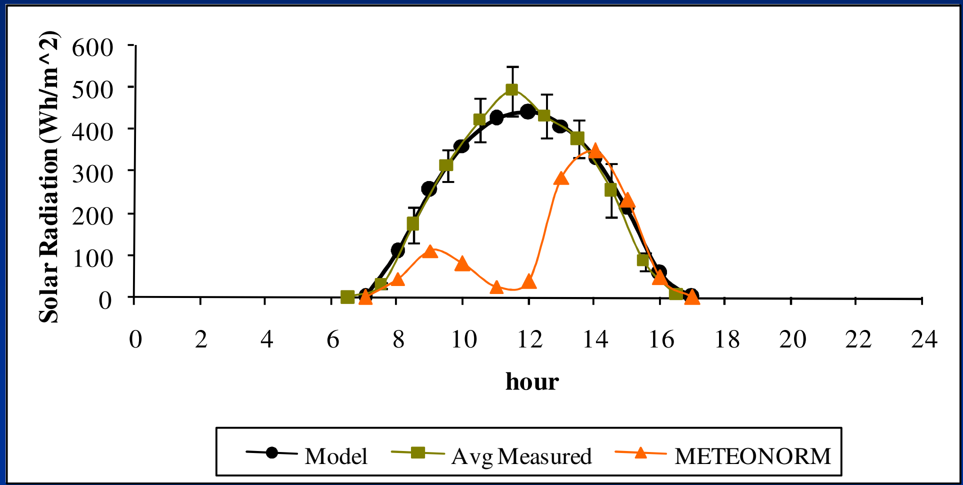
Thus, it provides the contribution to the $I(h;nj)$ prediction of the second derivative of $[I_{meas}(h;nj)- I_{av}(h;nj)]$, with respect to h.
R1 and R2 are random numbers Gaussianly distributed with a mean equal to zero and standard deviation equal to 1.

A comparison of the predicted mean $I_{m,pr}(h;17)$ and the measured $I_{mes}(h;17)$ values for the 17^{th} January and 17^{th} July

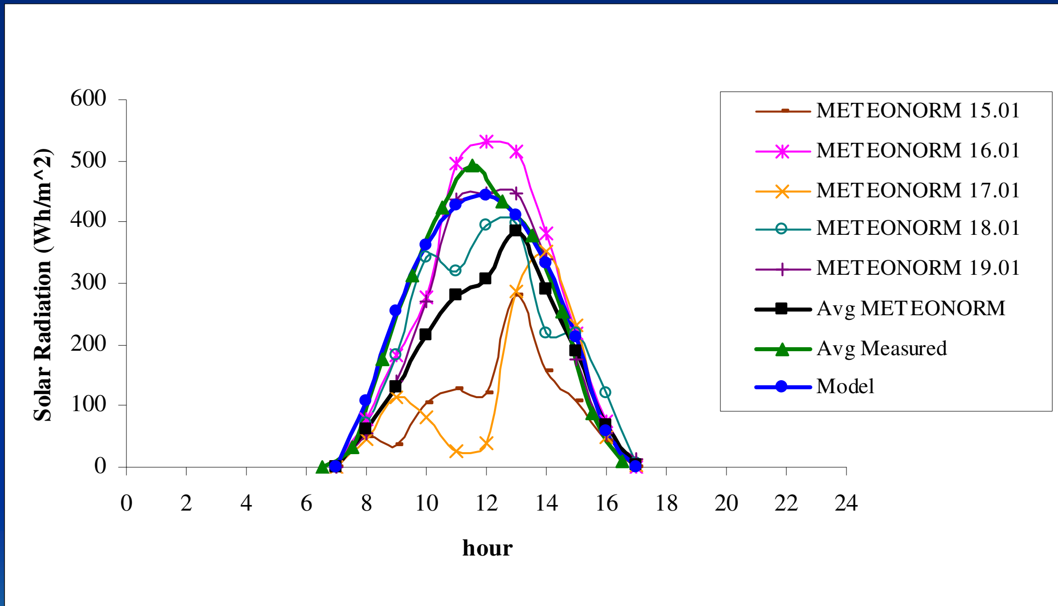




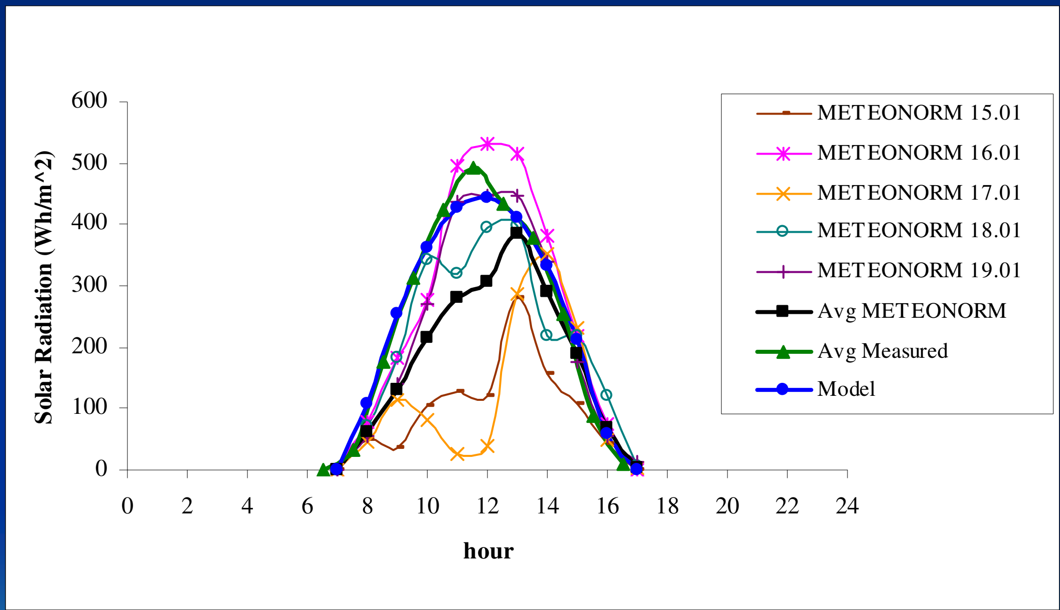
Values of the means for $I_{mes}(h;17)$, $I_{m,pr}(h;17)$ and $I_{MET}(h;17)$ for the 17th January



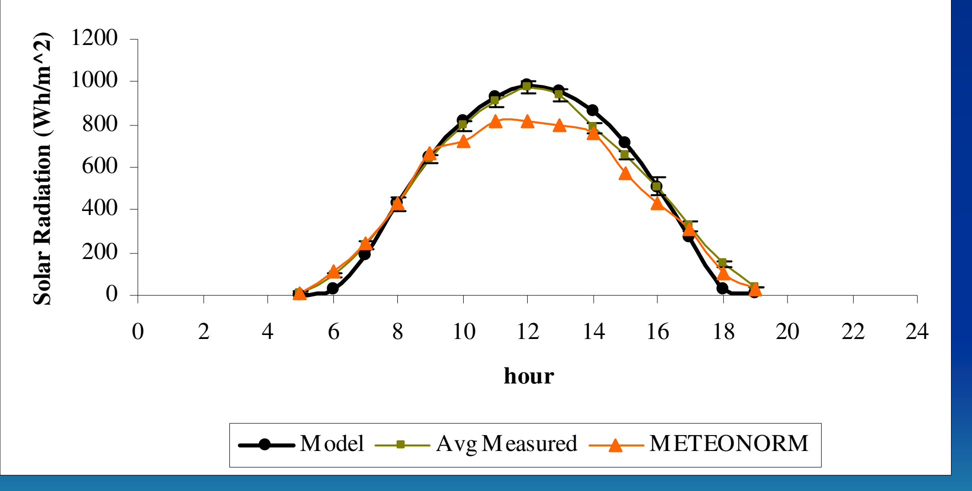
A comparison between the predicted $I_{MET}(h;n_j)$ values by METEONORM for the days 15th-19th January and, consequently, their average values on one hand, and the predicted corresponding values $I_{m,pr}(h;n_j)$ by this model, with reference to the average measured $I_{mes}(h;n_j)$ values.



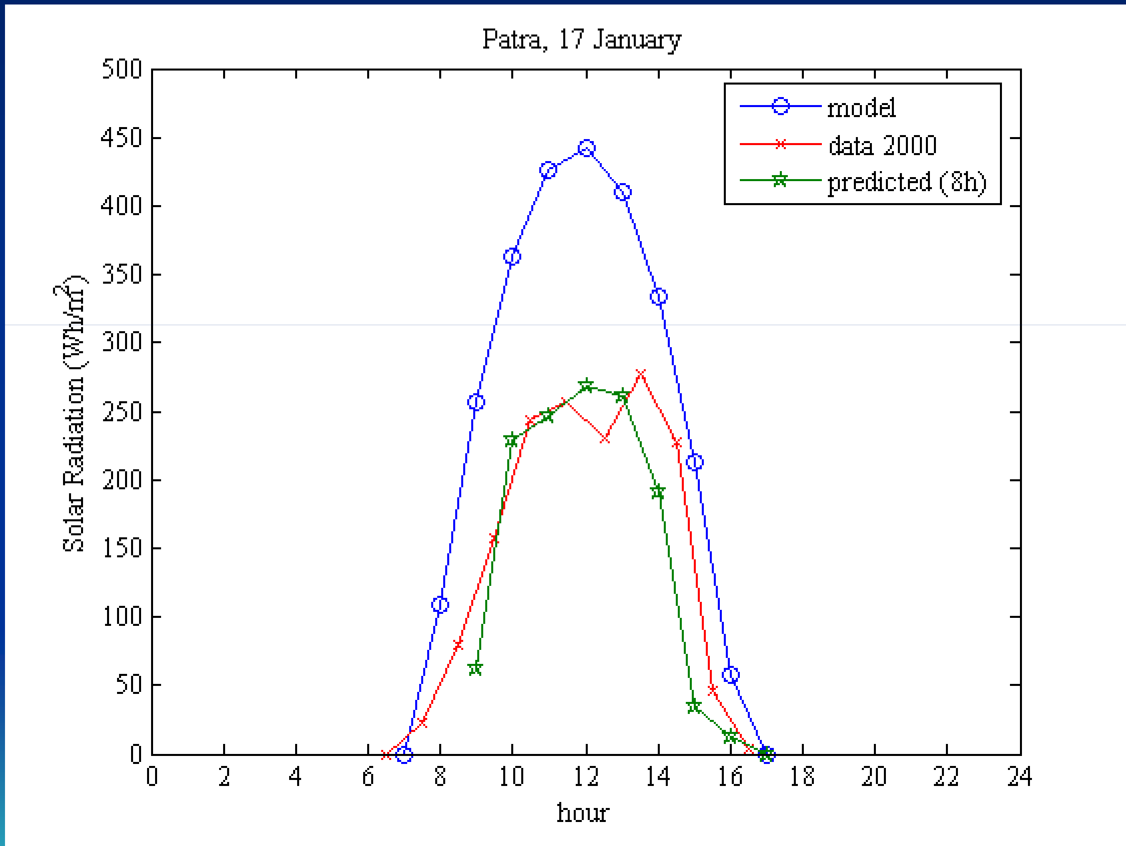
A comparison between the predicted $I_{MET}(h;n_j)$ values by METEONORM for the days 15th-19th January and, consequently, their average values on one hand, and the predicted corresponding values $I_{m,pr}(h;n_j)$ by this model, with reference to the average measured $I_{mes}(h;n_j)$ values.



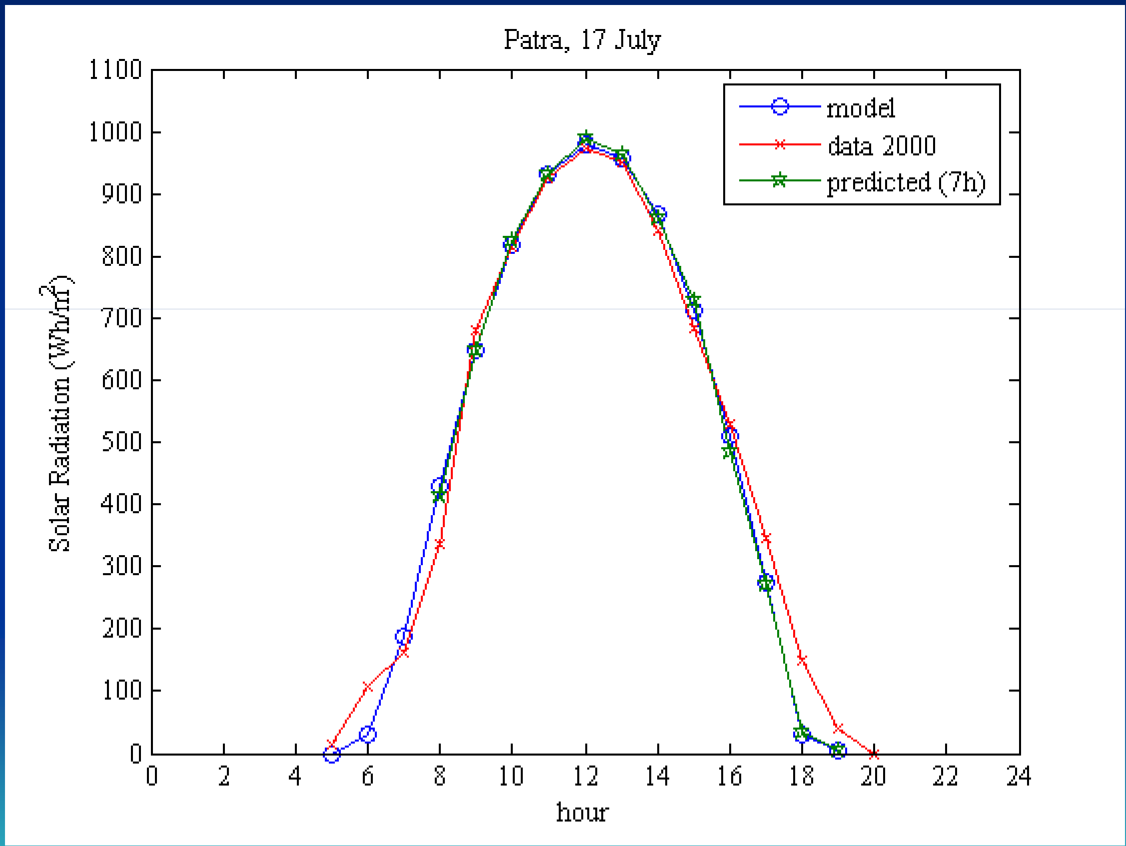
Values of the means for $I_{mes}(h;198)$, $I_{m,pr}(h;198)$ and $I_{MET}(h;198)$ for the 17th July



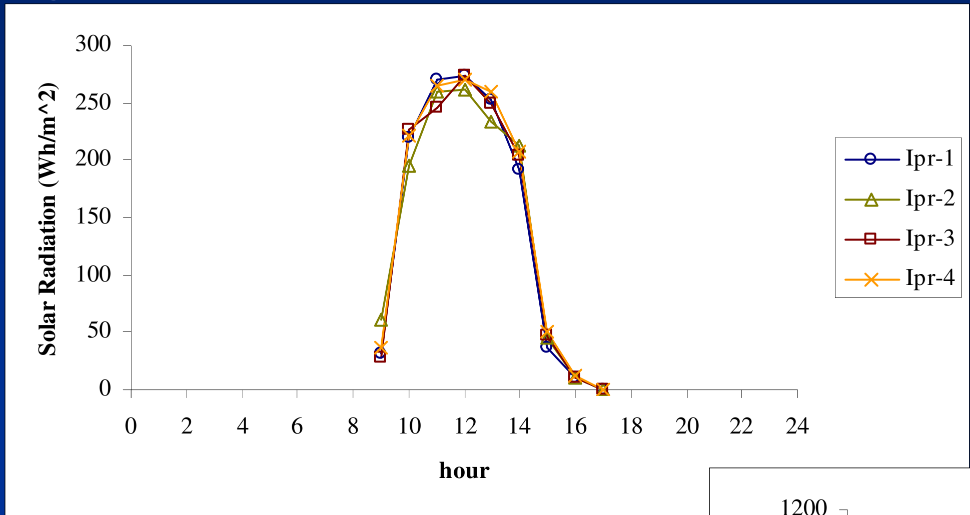
Mean predicted hourly global solar radiation values, $I_{m,pr}(h;17)$; the measured ones, $I_{mes}(h;17)$ for the 17th January 2000 and the predicted $I_{pr}(h;17)$ values, based on a single morning measurement at 8h

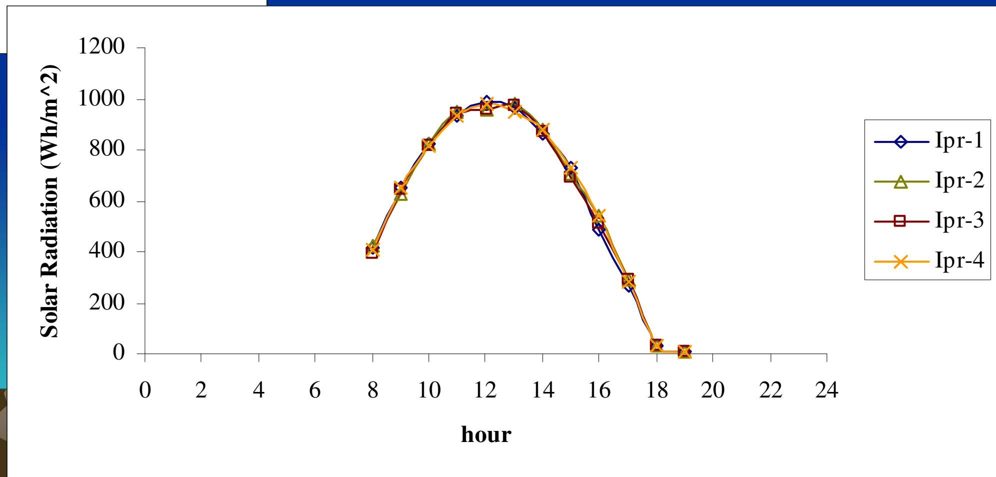


Mean predicted hourly global solar radiation values, $I_{m,pr}(h;198)$; the measured ones, $I_{mes}(h;198)$ for the 17th July 2000 and the predicted $I_{pr}(h;198)$ values, based on a single morning measurement at 7h

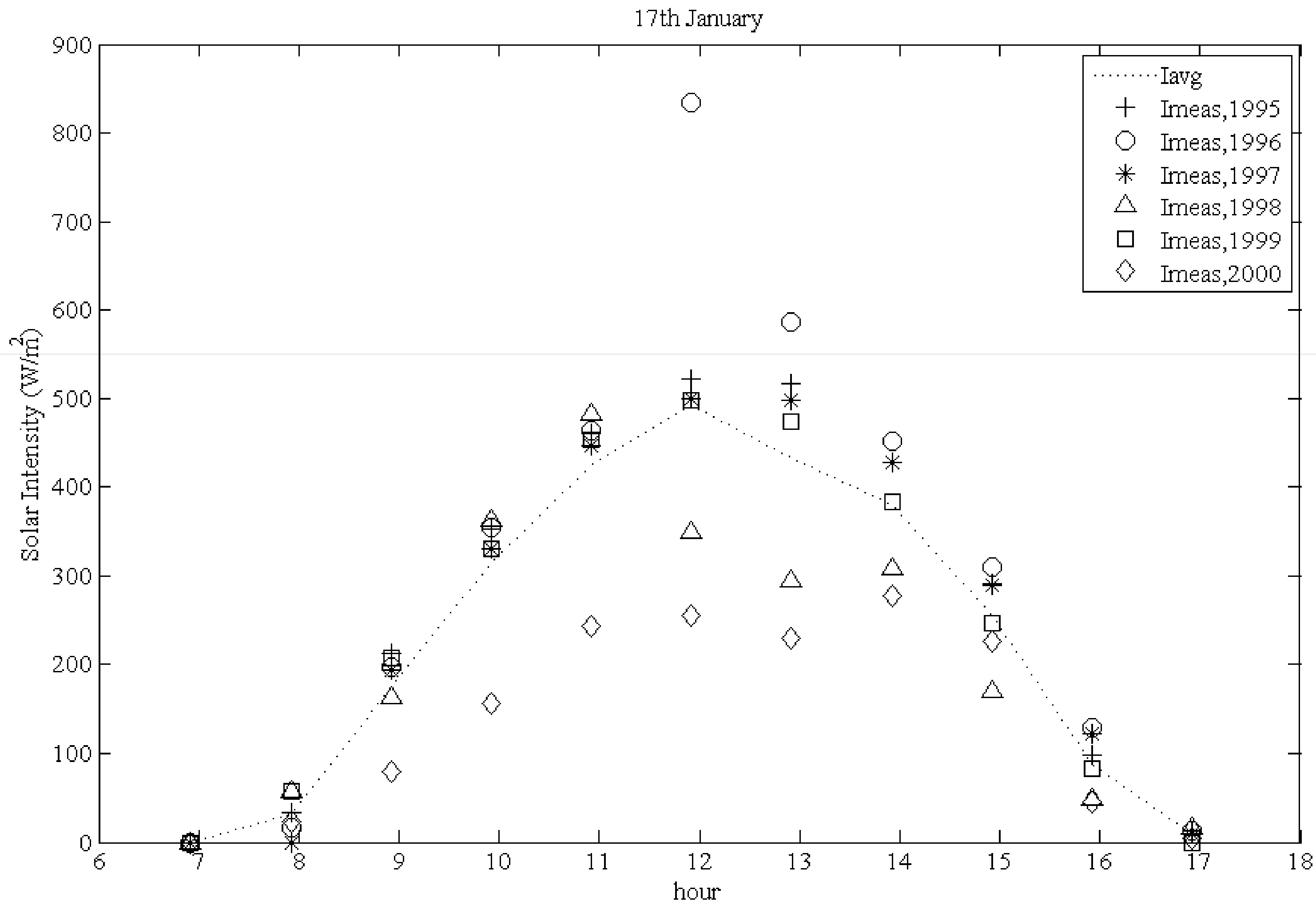


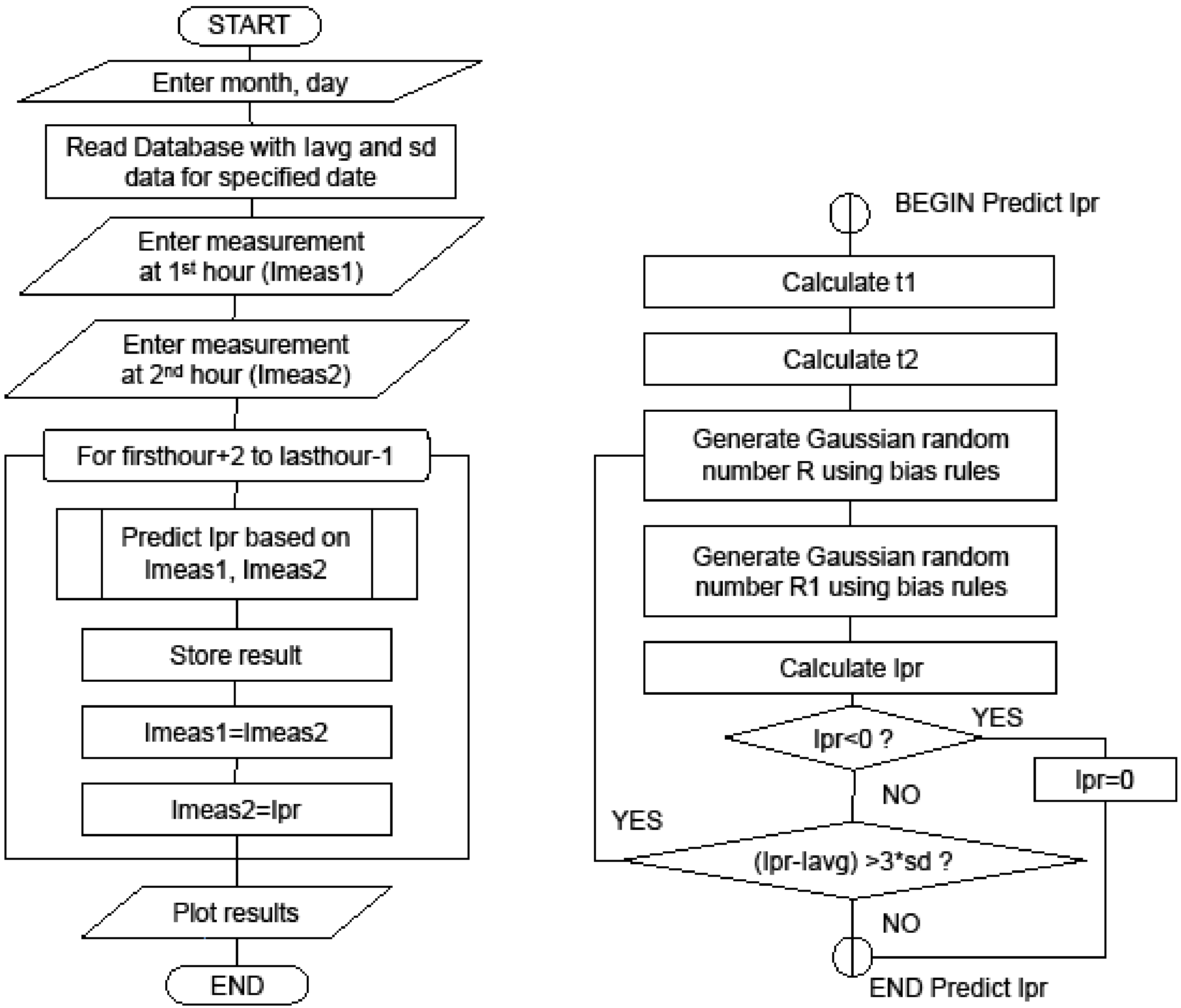
Four runs (series Ipr-1 to -4) of the daily $I_{pr}(h;17)$ values based on
the measured $I_{mes}(8;17)$ value for the 17th January and the 8h
and the measured $I_{mes}(7;198)$ value for the 17th July and the 7h





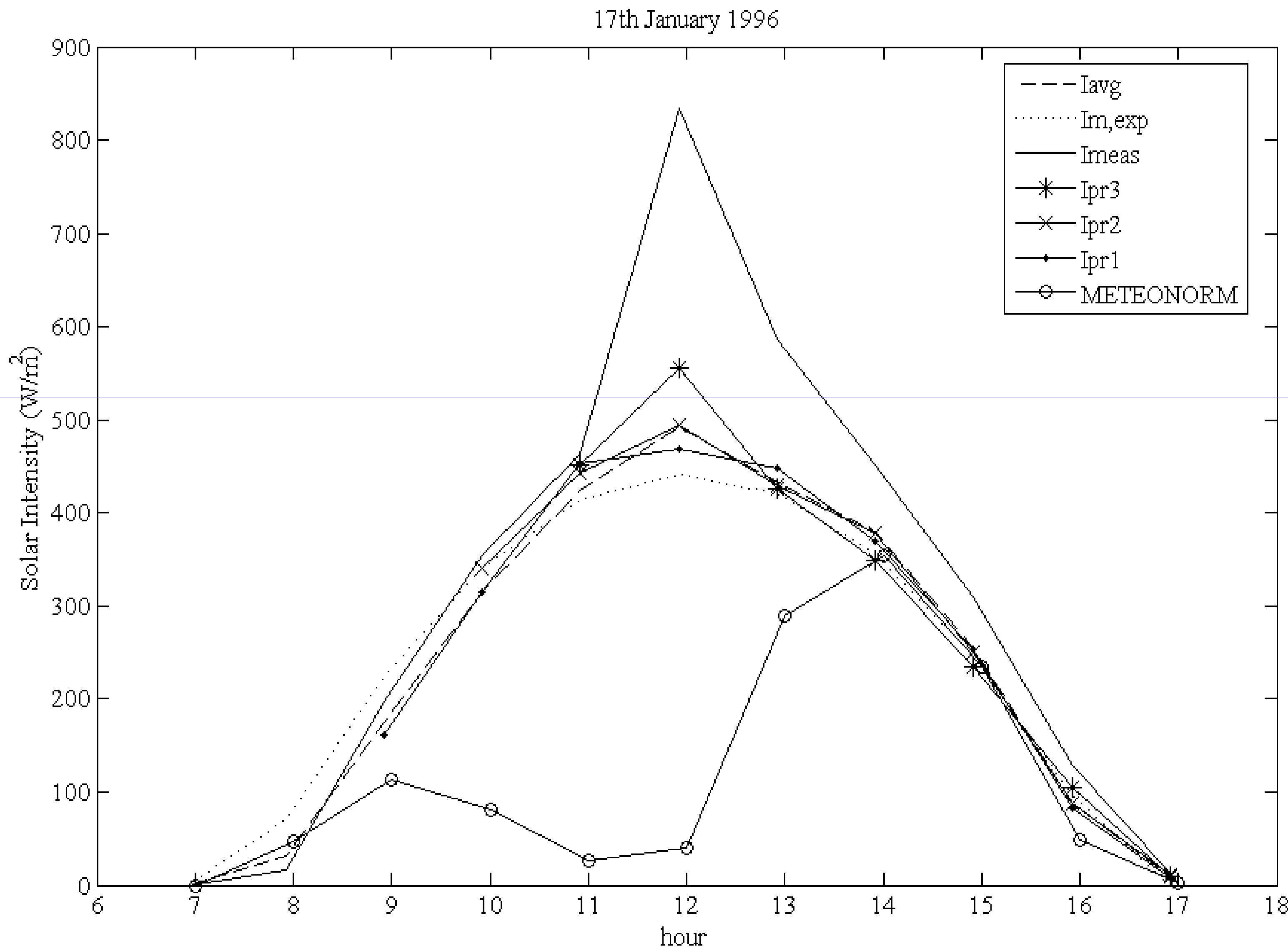
Measured horizontal global solar radiation intensity $I_{meas}(h;n_j)$, for the 17th January, for Patra, Greece, and for the years 1995-2000, and average hourly profile $I_{av}(h;n_j)$.



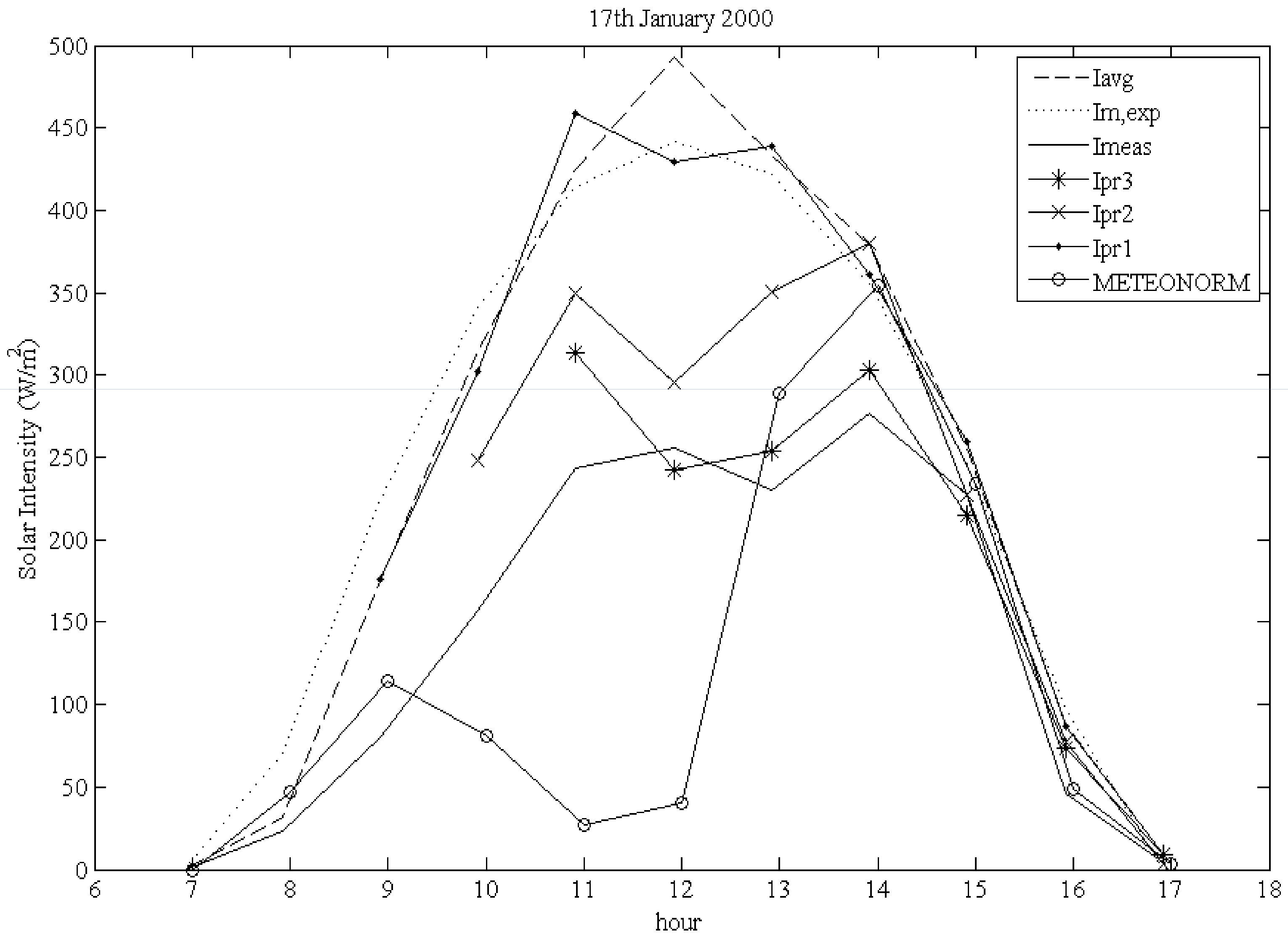


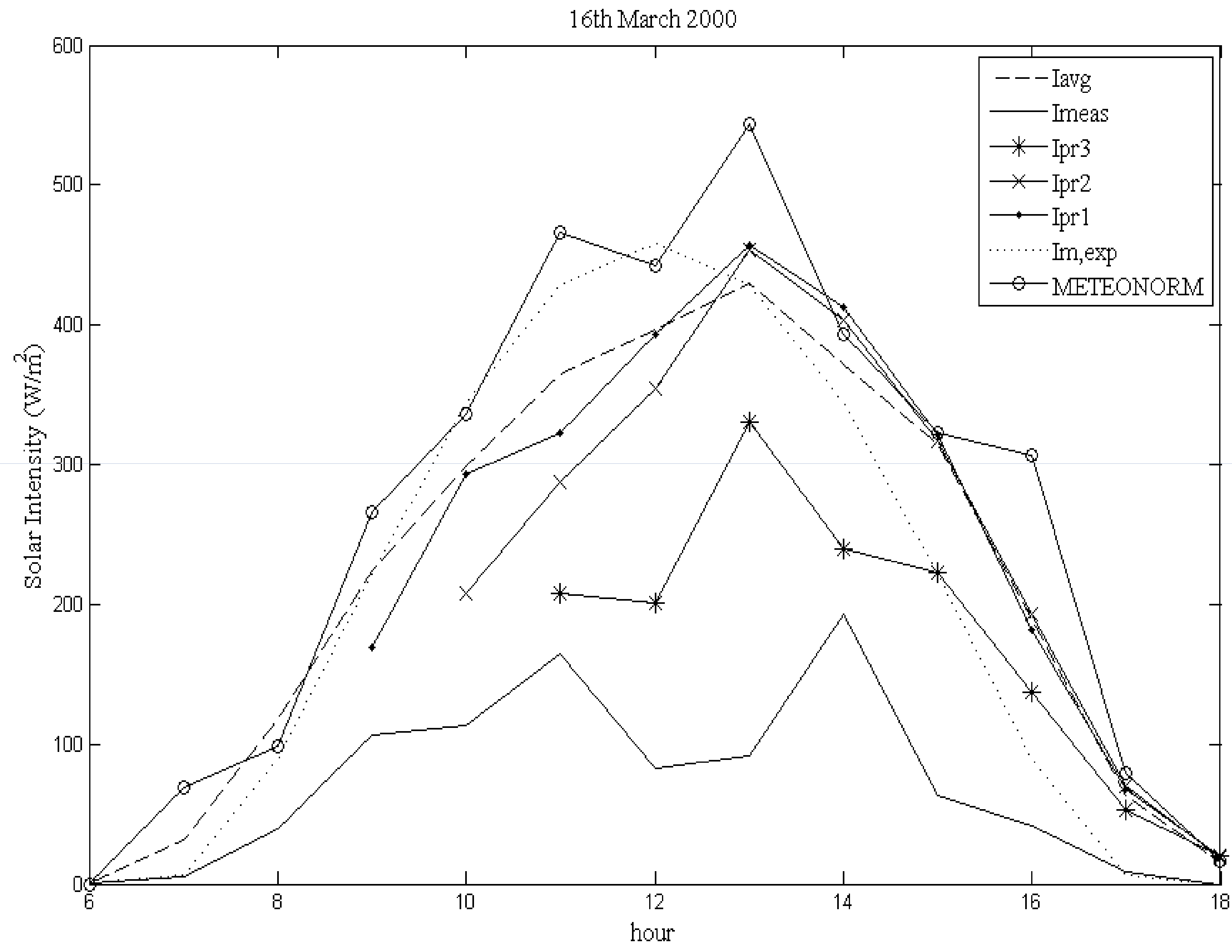
START
Enter month, day
Read Database with Iavg and sd data for specified date
Enter measurement at 1st hour (Imeas1)
Enter measurement at 2nd hour (Imeas2)
For firsthour+2 to lasthour-1
Predict Ipr based on Imeas1, Imeas2
Store result
Imeas1=Imeas2
Imeas2=Ipr
Plot results
END
BEGIN Predict Ipr
Calculate t1
Calculate t2
Generate Gaussian random number R using bias rules
Generate Gaussian random number R1 using bias rules
Calculate Ipr
Ipr<0 ?
YES
Ipr=0
NO
YES
(Ipr-Iavg) >3*sd ?
NO
END Predict Ipr

Prediction Results

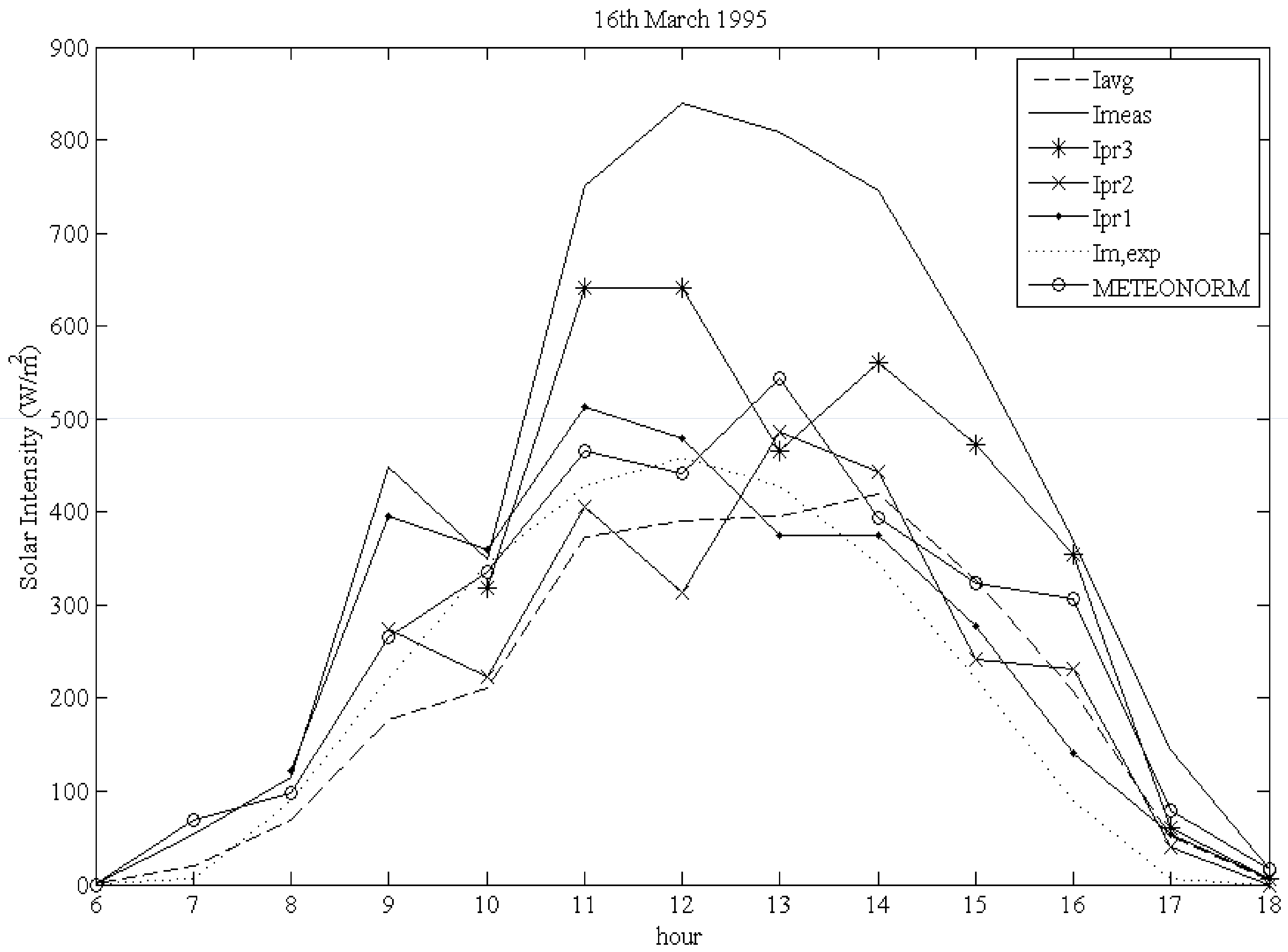


Prediction Results

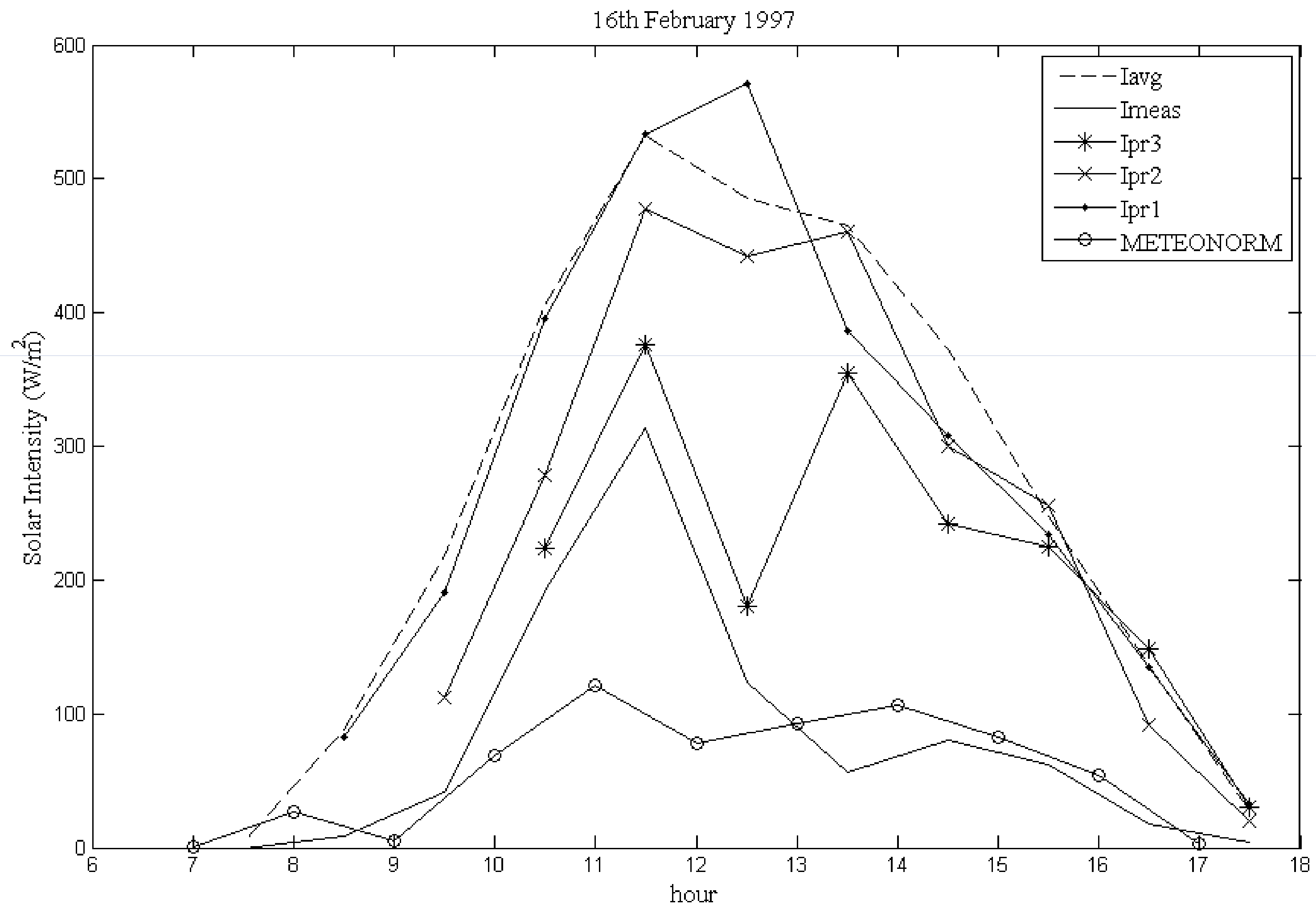



16th March 2000
Solar Intensity (W/m2)
hour
0
100
200
300
400
500
600
6
8
10
12
14
16
18
Iavg
Imeas
Ipr3
Ipr2
Ipr1
Im,exp
METEONORM

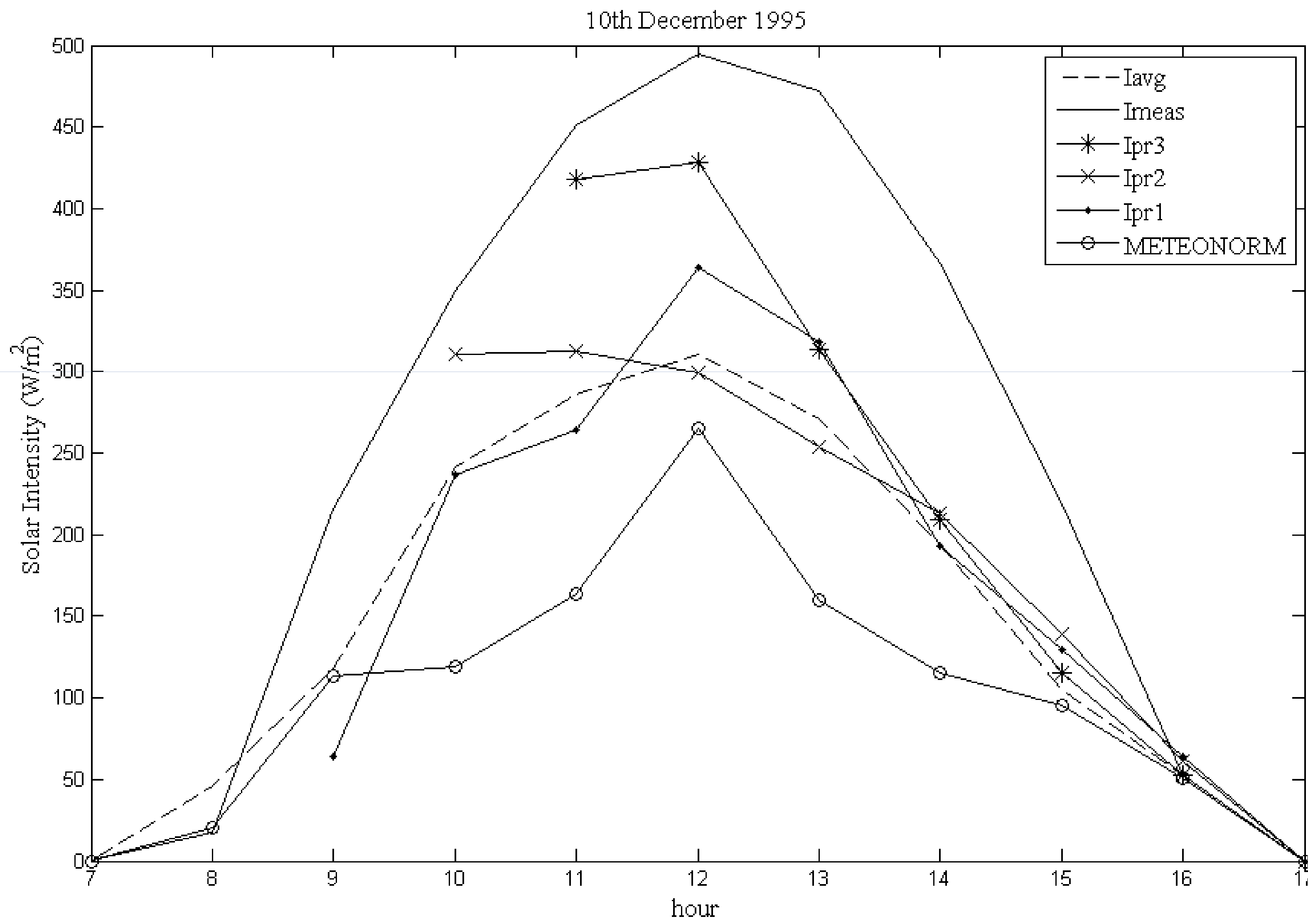
Prediction Results



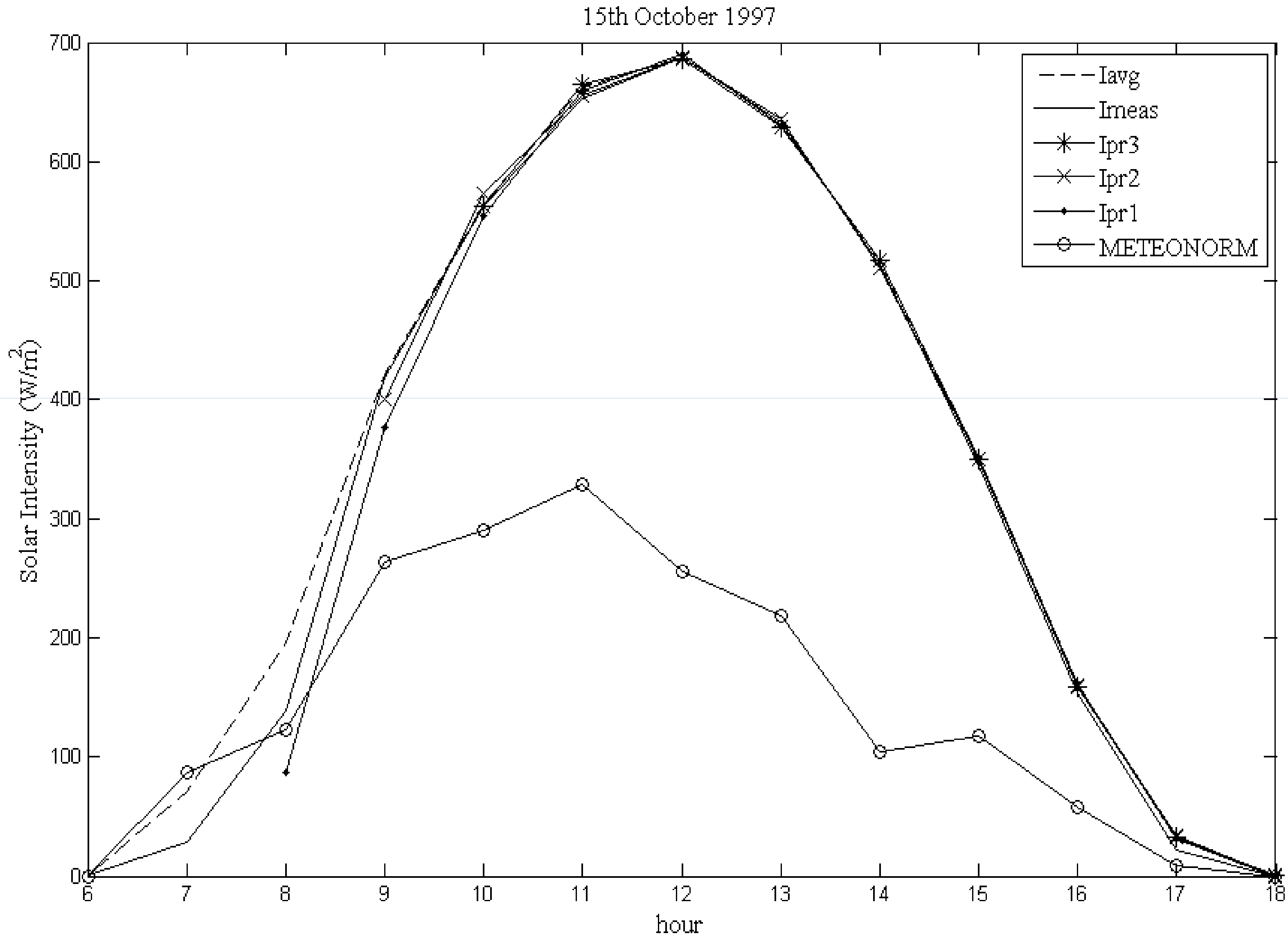
Prediction Results

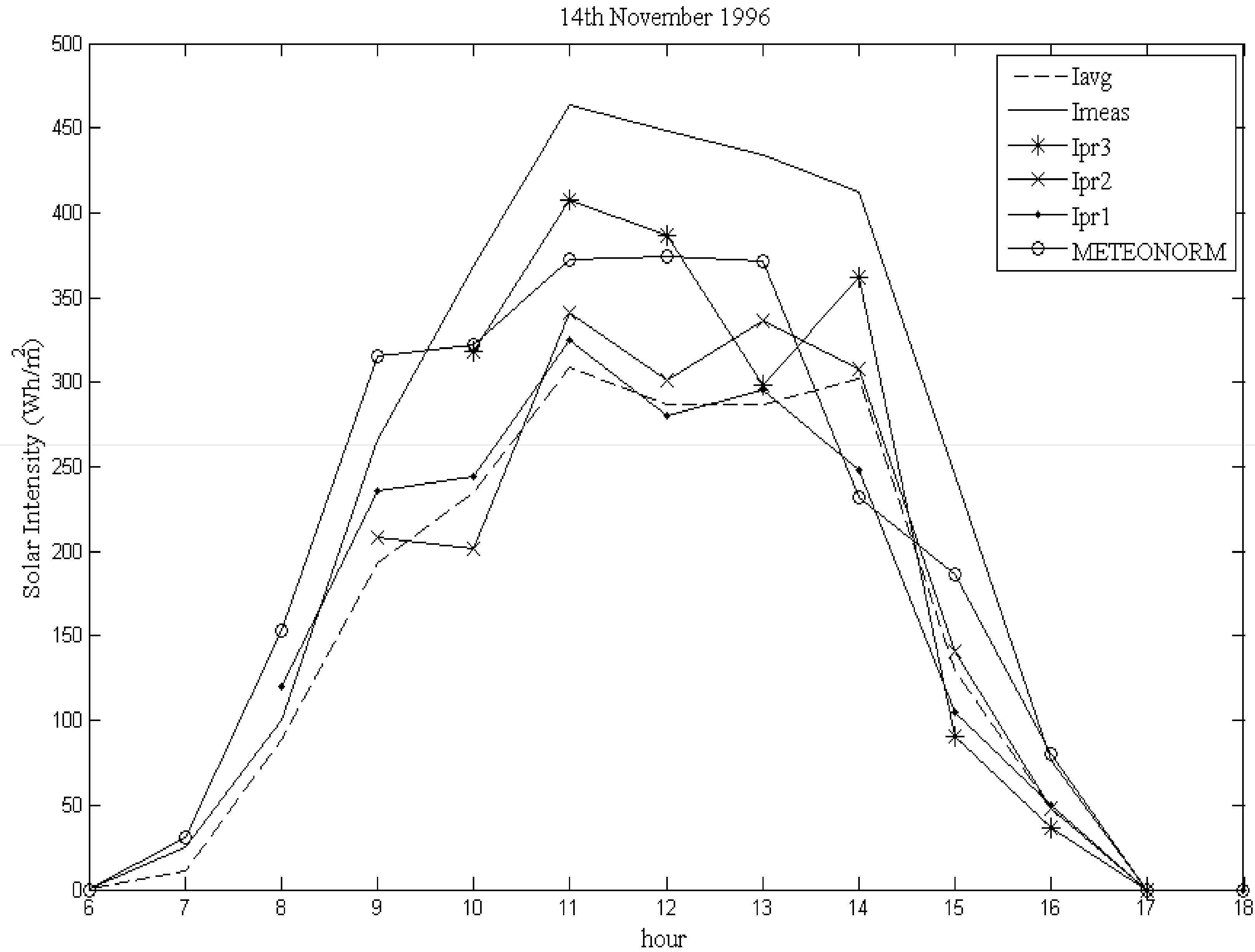


Prediction Results



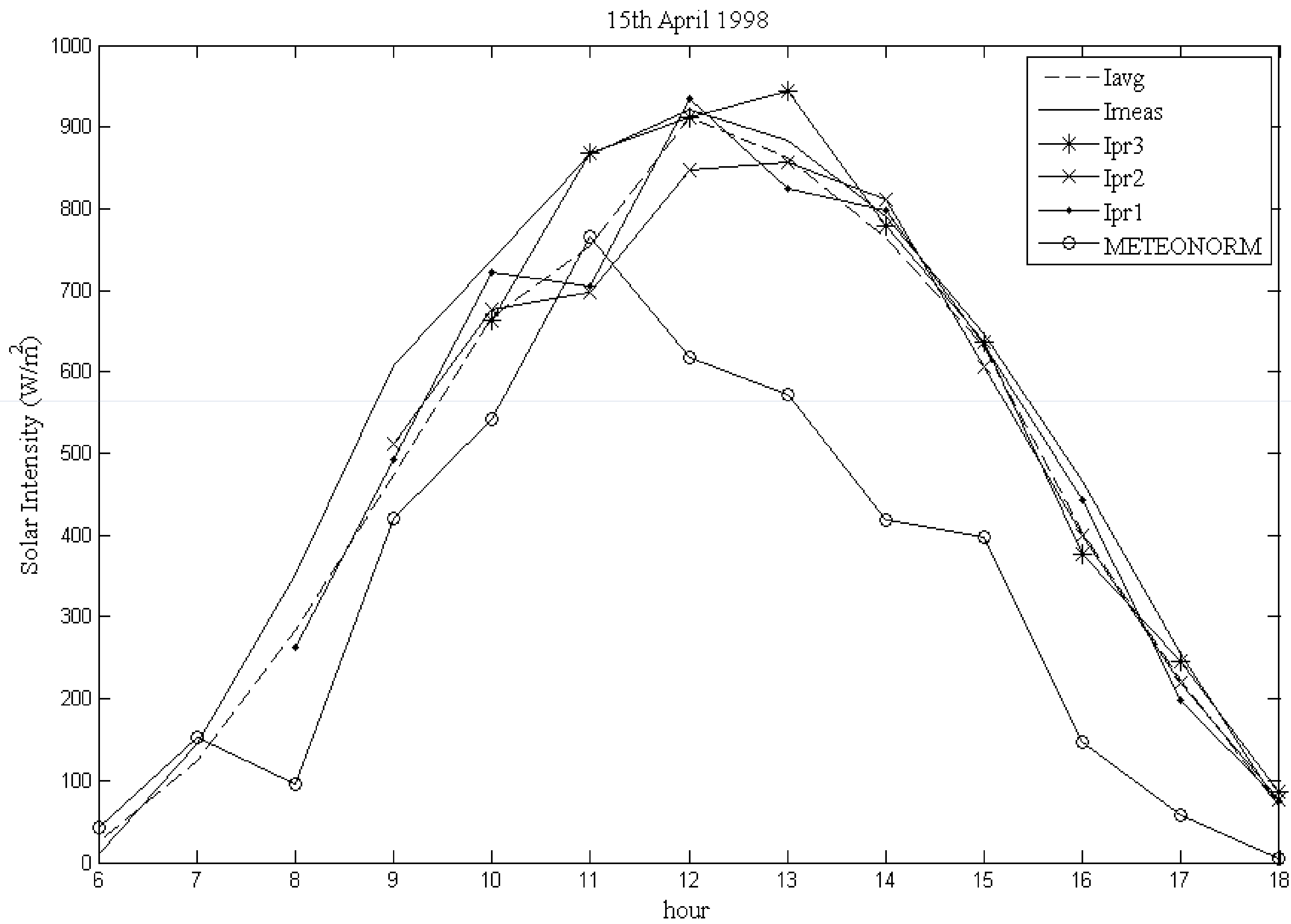
Prediction Results





14th November 1996
Solar Intensity (Wh/m²)
hour
Iavg
Imeas
Ipr3
Ipr2
Ipr1
METEONORM
0
50
100
150
200
250
300
350
400
450
500
6
7
8
9
10
11
12
13
14
15
16
17
18

Prediction Results



Discussion

- The model outlined is based on the assumption that the relative position of $I_{meas}(h;nj)$ with respect to $I_{m,exp}(h;nj)$ or with respect to $I_{av}(h;nj)$ may not change more than $\pm 1^{*}\sigma_{I}$ per hour, for mild climates.
- Generally, Mode I provides good estimates of $I(h;n_j)$.
- To compare the 3 modes, cases were taken for the rep. days of January and March, when strong solar radiation fluctuations occur.
- In some cases, the I(h;nj) prediction by Mode I differs significantly from the measured values.
- The comparison was also extended for the representative days of February, April, May, October, November and December.

- Comparison shows that $I_{pr}(h;nj)$ by Mode II lie generally closer to the measured values than for Mode I, especially in cases where the level of $I_{meas}(h;nj)$ values in morning hours lies far from the mean expected $I_{m,exp}(h;nj)$ and the pattern of the differences $[I_{meas}(h;nj) - I_{m,exp}(h;nj)]$ undergoes fluctuations within $\pm 1^* \sigma_I(h;nj)$.
- This improvement is brought in by the 3rd factor in
- In general, Mode II provides to a good degree the shape or the I(h;nj) profile in the majority of the cases examined.
- It is only the high peaked profile shown and the case where morning fluctuations are large and far away from the average values, which are not predicted at a good estimate by this mode.

- Mode III of this model gives much better results compared to the other modes, and provides good profiles even in cases where I(h;nj) shows higher degree of fluctuations, as shown for the months November, December, January, February and March.

- The improvement is attributed to the fourth term, which takes into account the second derivative of the difference $[I_{meas}(hi;n_j) - I_{avg}(hi;n_j)]$ for the previous hours.

Conclusions

- A program was developed in MATLAB to simulate solar radiation fluctuations and implement a stochastic model generating the $I_{pr}(h;nj)$ hourly profile of the global solar radiation to occur in a day, based on corresponding morning measurements.
- The predicted profiles were compared to the measured values and Modes II and III gave predictions closer to the measurements than Mode I.
- Especially for months where high fluctuations occurred, Mode III gave the best results. The proposed model was found to provide reliable results for the I(h;nj) profile, for any execution of the program.

- This has a straight impact to the effective prediction of the Power/Energy to be delivered by a PV cell during a day

- which, then, enables the engineer to manage power sources and loads to a much better cost-effective sizing

References

A stochastic simulation model for reliable PV system sizing providing for solar radiation fluctuations
Kaplani, E. , Kaplanis, S.
Department of Mechanical Engineering, T.E.I. of Patras, Meg. Alexandrou 1, Koukouli 26334, Patra, Greece
Applied Energy , Volume 97, September 2012, Pages 970-981

Stochastic prediction of hourly global solar radiation for Patra, Greece
Kaplanis, S. , Kaplani, E.
Mechanical Engineering Dept., T.E.I. of Patras, Meg. Alexandrou 1, Patra, Greece
Applied Energy , Volume 87, Issue 12, December 2010, Pages 3748-3758

A model to predict expected mean and stochastic hourly global solar radiation I(h;nj) values
Kaplanis, S. , Kaplani, E.
Renewable Energy Laboratory, T.E.I. of Patra, Meg. Alexandrou 1, Patra 26334, Greece
Renewable Energy, Volume 32, Issue 8, July 2007, Pages 1414-1425

New methodologies to estimate the hourly global solar radiation; Comparisons with existing models
Kaplanis, S.N.
Renewable Energy ,Volume 31, Issue 6, May 2006, Pages 781-790