Stochastic prediction of hourly global solar radiation profiles

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Abstract

• A stochastic prediction model of the hourly profile of the $I(h;n_j)$ for any day $n_j$ at a site is outlined.
• It requires 1, 2, or 3 measurements of the global solar radiation in a day $n_j$, uses a D.B. and gives $I(h;n_j)$ for the rest hours.
• The model is validated against solar measurements.
• Conclusions are deducted for the predictive power of the model developed in MATLAB.
• It provides $I(h;n_j)$ profile predictions very close to the measured values and
• offers itself as a promising tool for a predictive on-line daily load management.
Introduction

• The prediction of the global solar radiation $I(h;n_j)$ on an hourly, $h$, basis, for any day, $n_j$, was the target of many attempts internationally.

• Review papers outline and compare the mean expected, $I_{m,\text{exp}}(h;n_j)$, values which are the results of several such significant approaches.

• A reliable methodology to predict the $I(h;n_j)$ profile, based on few data, taking into account morning measurement(s) and simulating the statistics of $I(h;n_j)$, is a challenge.
A model to predict close to reality $I(h;n_j)$ values would be useful in problems such as:

1. Meteo purposes
2. Sizing effectively and reliably the solar power systems i.e. PV generators
3. Management of solar energy sources, i.e. the output of the PV systems, as affected by the meteo conditions in relation to the power loads to be met.

The above issues drive the research activities towards the development of an improved effective methodology to predict the $I(h;n_j)$ for any day, $n_j$, of the year at any site with latitude $\varphi$. 
• One of those methodologies to predict the mean expected hourly global solar radiation, as proposed by the authors, provided a simple approach model based on the function:

\[ I(h; n_j) = a + b \cdot \cos\left(2 \cdot \pi \cdot \frac{h}{24}\right) \]  \hspace{1cm} (1)

where, \( \alpha \) and \( b \) are constants, which depend on the day \( n_j \) and the site \( \phi \),
• This model, overestimates $I(h;\eta_j)$ at the early morning and late afternoon hours, while it underestimates $I(h;\eta_j)$ around the solar noon hours.

• Although, the $I_{pr}(h;\eta_j)$ values fall, in general, within the range of the measured $I_{mes}(h;\eta_j)$ fluctuations, a more accurate and dynamic model had to be developed.

• That model should have inbuilt statistical fluctuations, like the METEONORM package, but with a more effective prediction power.

• A comparison of the $I_{pr}(h;\eta_j)$ values between METEONORM and the generalised model versions, outlined, in contrast to measured $I_{mes}(h;\eta_j)$ values, during the years (1995-2000), will be highlighted.
Basic Theoretical Analysis

- Due to the mentioned drawback of the simple model, a correction factor is introduced, normalized at solar noon

\[
\frac{e^{-\mu(n_j) \cdot x(\theta_z)}}{e^{-\mu(n_j) \cdot x(\theta_z; \omega = 0)}}
\]

\[
\mu(n_j) = -\ln \left( \frac{\frac{H(n_j)}{H_{ext}(n_j)}}{x_m(\theta_z)} \right)
\]
\[ H(n_j) = A + B \cdot \cos \left( \frac{2 \cdot \pi}{364} \cdot n_j + C \right) \]

<table>
<thead>
<tr>
<th>Zone</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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<tr>
<td>A</td>
<td>16.60</td>
<td>15.86</td>
<td>15.14</td>
<td>14.89</td>
<td>14.28</td>
<td>13.74</td>
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</tbody>
</table>

For Zone 1, \( B = 9.731 \), \( C = 9.367 \), \( A = 16.60 \).
• $x(\theta_z)$ is the distance the solar beam travels within the atmosphere and $x(\theta_z;\omega=0)$ is the distance at solar noon, for which $\omega=0$. Notice that,

$$x(\theta_z) = -R_g \cdot \cos(\theta_z) + \sqrt{R_g^2 \cdot \cos^2(\theta_z) + (R^2 - R_g^2)}$$

$$R = R_g + H_{atm}$$

where, $R_g$ is the earth’s radius = 6.35·10³ km, and $H_{atm}$ the height of the atmosphere =2.5 km.

Also, $\cos(\theta_Z)$ is given by formula below, where $\delta$ is the solar declination and $\omega$ is the hour angle.
\[ \cos(\theta_z) = \cos(\phi) \cdot \cos(\delta) \cdot \cos(\omega) + \sin(\phi) \cdot \sin(\delta) \]

For \( \omega = 0 \) or \( h=12 \), at solar noon, we get:

\[ \cos(\theta_{z,0}) = \cos(\phi) \cdot \cos(\delta) + \sin(\phi) \cdot \sin(\delta) = \cos(\phi - \delta) \]

\[ \theta_{z,0} = \phi - \delta \]

Notice that, at sunset \( \theta_z = 90^\circ \), and therefore, for sunset we:

\[ x(\theta_z = 90^\circ) = x(\omega_{ss}) = \sqrt{R^2 - R_g^2} \]
• $x_m(\theta_z)$ is the mean daily distance the solar beam travels in the atmosphere. It is determined by the formula:

$$x_m(\theta_z) = \frac{\sum_{i=1}^{n} x(\theta_z)_i}{N}$$

$$x_m(\theta_z) = \frac{\sum \left( x(\theta_z) \cdot I(h; n_j) \right)_i}{\sum I(h; n_j)_i}$$

More accurately, $x_m(\theta_z)$ should be weighted over the solar intensity.
Finally, the model to predict the mean expected hourly global solar radiation now takes the form:
\[ I(h; n_j) = \alpha + b \cdot \frac{e^{-\mu(n_j) \cdot x(h)} \cdot \cos \left( \frac{2 \cdot \pi \cdot h}{24} \right)}{e^{-\mu(n_j) \cdot x(h=12)}} \]

\( \alpha \) and \( b \) are determined, using the boundary conditions to be highlighted below:

\[ I(h; n_j) = 0 \text{ at } h = h_{ss}, \text{ where } h_{ss} = 12h + \omega_{ss}/15^0/h \]

\[ \omega_{ss} = \cos \left( -\tan(\varphi) \cdot \tan(\delta) \right) \]

Integration of \( I(h,n_j) \) over a day, from \( \omega_{sr} \) to \( \omega_{ss} \), provides \( H(n_j) \) and via these 2 boundary conditions we may determine \( a \) and \( b \)
A comparison of the predicted mean $I_{m, pr}(h; 17.1)$ and the measured $I_{mes}(h; 17.1)$ values, during winter for Patra, Greece, for the years 1995-2000.
The means of $I_{mes}(h;nj)$, the $I_{m,pr}(h;nj)$ values, as predicted by this model, and the $I_{MET}(h;n_j)$ values, with the inbuilt stochastic generator for fluctuations, provided by the METEONORM package for the days 17.1 and 17.7, are shown below.

The results show the presence of expected strong fluctuations in $I(h;n_j)$ especially, for winter months.

Values of the mean $I_{mes}(h;17.1)$, $I_{m,pr}(h;17.1)$ and $I_{MET}(h;17.1)$ for Patra, Greece.
Values of the mean \( l_{\text{mes}}(h;17.7) \), \( l_{\text{m,pr}}(h;17.7) \) and \( l_{\text{MET}}(h;17.7) \) for Patra, Greece.
The prediction methodology, proposed, takes into account a first early $I(h; n_j)$ measurement, at hour $h_1$.

The model to predict $I(h; n_j)$, as described in this paper, introduces a stochastic factor, which takes into account the previous hour $I(h-1; n_j)$ value.

The steps followed to predict $I(h; n_j)$ based on a morning measurement, $I_{mes}(h; n_j)$, are outlined below:
1. Let $I_{m,pr}(h;n_j)$ is predicted as outlined before. Such values are, easily, obtained using a MATLAB program and algorithms, developed for this research project.

2. For $n_j=17$, solar intensity measurement, at hour $h_1$, provides $I_{mes}(h_1;17)$, whose s.d. is $\sigma_i$.

3. $\sigma_i$ is pre-determined for morning hours $[h_{sr}, h_{sr}+3]$, afternoon periods $[h_{ss}-3, h_{ss}]$, as well as for the hours around solar noon.

For example: Let the p.d.f. is a normal distribution with $\sigma/I*100\% = 25\%$ at morning hours.
4. Let us start with $I_{mes}(h_1;17)$. The program predicts $I_{m,pr}(h_1;17)$ and subtracts it from the measured $I_{mes}(h_1;17)$, for that hour, $h_1$. The result is compared to $\sigma_I$,

$$\frac{I_{mes}(h_1;17) - I_{m,pr}(h_1;17)}{\sigma_I} = \frac{\delta I}{\sigma_I} = \lambda$$

Possible values of the above expression for $\delta I/\sigma_I$, according to normal distribution may lie in the region: [-4, +4]. The interval of $\sigma_I$, where $I(h;nj)$ lies, in the first hour $h_1$, may be determined by equation above.

Let this deviation be $\lambda^*\sigma_I$. 
5. An attempt is made to predict $I_{pr}(h_2;n_j)$, at $h_2 = h_1 + 1$ hour. The model tries to give an estimate of the $I_{pr}(h_2;n_j)$, taking into account the deviation of $I_{mes}(h_1;17)$ from the mean expected value $I_{m,pr}(h_1;17)$, as determined in step 4.

In this step, the model samples from a Gaussian p.d.f. in order to estimate in which $\sigma'_i$ interval of the normal distribution, the $I(h_2;n_j)$ value lies. That is, it determines the new value of $\lambda^*\sigma_i$; let it be $\lambda'\cdot\sigma'_i$. 
6. The predicted value of $I(h_2;n_j)$ is given, in this step, based on the mean expected $I_{m,pr}(h_2;17)$, with a new deviation value $\lambda' \cdot \sigma_1'$, i.e.

$$I_{pr}(h_2;n_j) = I_{m,pr}(h_2;n_j) \pm \lambda' \cdot \sigma_1'$$

$\lambda'$ is determined through a Gaussian sampling and is permitted to take, according to this model, values within the range $\lambda \pm 1$.

7. The model, then, determines the $I_{pr}(h_2;n_j)$ value and compares it with the mean expected $I_{m,pr}(h_2;n_j)$. It repeats the cycle, steps 4-6, for the $h_3$ hour and so on.
It is important to note that if $\lambda$ is as extreme as +3 or -3, the model requires that $I_{pr}(h;n_j)$ lies in the same $\sigma_i$ region, for all day long, without jumping to other $\sigma_i$ intervals.
Patra, 17 July

- **Model**
- **Data 2000**
- **Predicted (7h)**

**Solar Radiation (Wh/m²)**

- Hour 0 to 24
- Y-axis: 0 to 1200
- X-axis: 0 to 24
From Mode I: $I(h;n_j)$ prediction based on one morning measurement to Mode II: $I(h;n_j)$ prediction based on two morning measurements

- Mode II of this model considers for the $I(h;n_j)$ prediction two morning solar radiation measurements.
- In addition to Mode I, it takes into account the rate of change of the difference $[I_{\text{meas}}(h;n_j) - I_{\text{av}}(h;n_j)]$, during the period from $h_1$ to $h_2$.
- Conclusively, it includes two stochastic terms, one term which stands for the stochastic fluctuations at hour $h_3$, and
- a second term to stand for the rate of change of the $I(h;n_j)$, within the time interval $[h_1, h_2]$. 
Similarly, as in Mode I, the parameter \( t_2 \) is determined:

\[
 t_2 = \frac{I_{\text{meas}}(h_2; n_j) - I_{\text{av}}(h_2; n_j)}{\sigma I(h_2; n_j)}
\]

Finally, the \( I(h_3;n_j) \) value for the next time interval, \( h_3 \), is predicted by an improved expression:

\[
 I_{pr}(h_3;n_j) = I_{av}(h_3;n_j) + R \cdot \sigma I(h_3;n_j) + \frac{1}{4} \left( t_2 \cdot \sigma I(h_2;n_j) - t_1 \cdot \sigma I(h_1;n_j) \right) \cdot R_1
\]
• $\sigma_1(h_2;n_j)$ and $\sigma_1(h_3;n_j)$ are the s.d. of the measured $I(h;n_j)$ values at hours $h_2$ and $h_3$, respectively, in the day $n_j$, as obtained from the D.B.

• Furthermore, the model proceeds to predict the $I_{pr}(h_4;n_j)$ value. This is based on $I_{pr}(h_3;n_j)$, which is the previous hour, $h_3$, predicted value and the measured $I_{meas}(h_2;n_j)$ one.

• Further on, it may predict $I_{pr}(h_5;n_j)$, based on the previously predicted values $I_{pr}(h_3;n_j)$ and $I_{pr}(h_4;n_j)$, and so on.
• Notice that, R is a random number which may take values within $t_2 \pm 1$.

• This implies that from hour to hour, one may not expect weather variations larger than $\pm 1*\sigma_1$

• $R_1$ is randomly distributed according to a Gaussian p.d.f. $(0, 1)$. The term $(t_2*\sigma_1(h_2;n_j) - t_1*\sigma_1(h_1;n_j))*R_1$ stands for the contribution to the $I(h;n_j)$ prediction by its rate of change during the 2 previous hours.

This contribution is estimated by the relative positions of the measured $I_{\text{meas}}(h_1;n_j)$ and $I_{\text{meas}}(h_2;n_j)$, with reference to the average values $I_{\text{av}}(h_1;n_j)$ and $I_{\text{av}}(h_2;n_j)$, one by one.
• The above Formula expresses the superposition principle of two processes.

• The first one is the short term stochastic behaviour which provides expected hourly fluctuations based on the past history of the stored data for the hour $h$ of a day $n_j$.

• The second one represents the present trend of the hourly $I(h;n_j)$ measurements during the interval $h_1$ to $h_2$. This trend is weighted over a Gaussian p.d.f., $(0, 1)$, which underlines that the two processes, short and long term, are independent to each other, and this ensemble represents the real phenomenon.
Mode III: $I(h;n_j)$ prediction based on three morning measurements

- The Mode III of the proposed model takes into consideration 3 morning solar global radiation measurements for the prediction of $I(h;n_j)$.

- According to the concept presented earlier, the prediction of $I(h_4;n_j)$ at hour $h_4$ is based on the following formula, which is more advanced than the previous two.
\[ I_{pr}(h_4;n_j) = I_{av}(h_4;n_j) + R \cdot \sigma_{I(h_4;n_j)} + \frac{1}{4} \left( t_3 \cdot \sigma_{I(h_3;n_j)} - t_2 \cdot \sigma_{I(h_2;n_j)} \right) \cdot R1 + \]
\[ + \frac{1}{9} \left( t_3 \cdot \sigma_{I(h_3;n_j)} - 2 \cdot t_2 \cdot \sigma_{I(h_2;n_j)} + t_1 \cdot \sigma_{I(h_1;n_j)} \right) \cdot R2 \]

In eq above there appears a 3\(^{rd}\) and 4\(^{th}\) term.
The 3\(^{rd}\) term gives a measure of the rate of change of \([I_{meas}(h;n_j)-I_{av}(h;n_j)]\), during the two hours, \([h_2, h_3]\), prior to the hour, \(h_4\).

The fourth term gives the rate of change of the above difference during the three previous hours.

Thus, it provides the contribution to the \(I(h;n_j)\) prediction of the second derivative of \([I_{meas}(h;n_j)-I_{av}(h;n_j)]\), with respect to \(h\).

R1 and R2 are random numbers Gaussianly distributed with a mean equal to zero and standard deviation equal to 1.
A comparison of the predicted mean $I_{m,pr}(h;17)$ and the measured $I_{mes}(h;17)$ values for the 17\textsuperscript{th} January and 17\textsuperscript{th} July.
Values of the means for $I_{\text{mes}}(h;17)$, $I_{\text{m,pr}}(h;17)$ and $I_{\text{MET}}(h;17)$ for the 17th January
A comparison between the predicted $I_{\text{MET}}(h;n_j)$ values by METEONORM for the days 15th-19th January and, consequently, their average values on one hand, and the predicted corresponding values $I_{m,\text{pr}}(h;n_j)$ by this model, with reference to the average measured $I_{\text{mes}}(h;n_j)$ values.
A comparison between the predicted $I_{\text{MET}}(h; n_j)$ values by METEONORM for the days 15th-19th January and, consequently, their average values on one hand, and the predicted corresponding values $I_{\text{m,pr}}(h; n_j)$ by this model, with reference to the average measured $I_{\text{mes}}(h; n_j)$ values.
Values of the means for $I_{\text{mes}}(h;198)$, $I_{m,pr}(h;198)$ and $I_{\text{MET}}(h;198)$ for the 17$^{th}$ July.
Mean predicted hourly global solar radiation values, $I_{m,pr}(h;17)$; the measured ones, $I_{mes}(h;17)$ for the 17th January 2000 and the predicted $I_{pr}(h;17)$ values, based on a single morning measurement at 8h.
Mean predicted hourly global solar radiation values, $I_{m,pr}(h;198)$, the measured ones, $I_{mes}(h;198)$ for the 17th July 2000 and the predicted $I_{pr}(h;198)$ values, based on a single morning measurement at 7h
Four runs (series Ipr-1 to -4) of the daily $I_{pr}(h;17)$ values based on the measured $I_{mes}(8;17)$ value for the 17th January and the 8h and the measured $I_{mes}(7;198)$ value for the 17th July and the 7h.
Measured horizontal global solar radiation intensity $I_{\text{meas}}(h;n_j)$, for the 17th January, for Patra, Greece, and for the years 1995-2000, and average hourly profile $I_{\text{av}}(h;n_j)$. 
START
Enter month, day
Read Database with lavg and sd data for specified date
Enter measurement at 1st hour (I meas1)
Enter measurement at 2nd hour (I meas2)
For firsthour+2 to lasthour-1
Predict Ipr based on I meas1, I meas2
Store result
I meas1=I meas2
I meas2=Ipr
Plot results
END

BEGIN Predict Ipr
Calculate t1
Calculate t2
Generate Gaussian random number R using bias rules
Generate Gaussian random number R1 using bias rules
Calculate Ipr
Ipr<0 ?
YES
Ipr=0
NO
(lpr-lavg) >3*sd ?
YES
NO
END Predict Ipr
Prediction Results

17th January 1996

Solar Intensity (W/m²)

hour

Legend:
- Iavg
- Irr,exp
- I meas
- Ipr3
- Ipr2
- Ipr1
- METEONORM
Prediction Results

[Graph showing solar intensity over time on 16th February 1997]
Prediction Results

10th December 1995

- Iavg
- Imeas
- Ipr3
- Ipr2
- Ipr1
- METEONORM

Solar Intensity (W/m²) vs. hour
Prediction Results

15th October 1997

Solar Intensity (W/m²) vs. hour

- $I_{avg}$
- $I_{meas}$
- $I_{pr3}$
- $I_{pr2}$
- $I_{pr1}$
- METEONORM
Prediction Results

15th April 1998

Solar Intensity (W/m²)

hour
Discussion

• The model outlined is based on the assumption that the relative position of $I_{\text{meas}}(h;n_j)$ with respect to $I_{m,\text{exp}}(h;n_j)$ or with respect to $I_{\text{av}}(h;n_j)$ may not change more than $\pm 1*\sigma_I$ per hour, for mild climates.

• Generally, Mode I provides good estimates of $I(h;n_j)$.

• To compare the 3 modes, cases were taken for the rep. days of January and March, when strong solar radiation fluctuations occur.

• In some cases, the $I(h;n_j)$ prediction by Mode I differs significantly from the measured values.

• The comparison was also extended for the representative days of February, April, May, October, November and December.
• Comparison shows that \( I_{pr}(h; nj) \) by Mode II lie generally closer to the measured values than for Mode I, especially in cases where the level of \( I_{meas}(h; nj) \) values in morning hours lies far from the mean expected \( I_{m,exp}(h; nj) \) and the pattern of the differences \([I_{meas}(h; nj) – I_{m,exp}(h; nj)]\) undergoes fluctuations within \( \pm 1^* \sigma_I(h; nj) \).

• This improvement is brought in by the 3\textsuperscript{rd} factor in

• In general, Mode II provides to a good degree the shape or the \( I(h; nj) \) profile in the majority of the cases examined.

• It is only the high peaked profile shown and the case where morning fluctuations are large and far away from the average values, which are not predicted at a good estimate by this mode.
• Mode III of this model gives much better results compared to the other modes, and provides good profiles even in cases where \( I(h; nj) \) shows higher degree of fluctuations, as shown for the months November, December, January, February and March.

• The improvement is attributed to the fourth term, which takes into account the second derivative of the difference \( [I_{\text{meas}}(hi; nj) - I_{\text{avg}}(hi; nj)] \) for the previous hours.
Conclusions

• A program was developed in MATLAB to simulate solar radiation fluctuations and implement a stochastic model generating the $I_{pr}(h; nj)$ hourly profile of the global solar radiation to occur in a day, based on corresponding morning measurements.

• The predicted profiles were compared to the measured values and Modes II and III gave predictions closer to the measurements than Mode I.

• Especially for months where high fluctuations occurred, Mode III gave the best results. The proposed model was found to provide reliable results for the $I(h; nj)$ profile, for any execution of the program.
• This has a straight impact to the effective prediction of the Power/Energy to be delivered by a PV cell during a day

• which, then, enables the engineer to manage power sources and loads to a much better cost-effective sizing
• References

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